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Secondary

MATHEMATICS

Class-VII

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Contents

CHAPTER 1 RATIONAL NUMBERS

INTRODUCTION

Do you remember number systems?

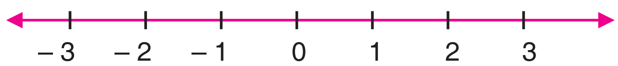
(i) Numbers 1, 2, 3, 4, . . . which we use for counting, form the **system of natural numbers**.

Natural numbers are
1, 2, 3, 4 ....

(ii) Natural numbers along with zero, form the **system of whole numbers**.

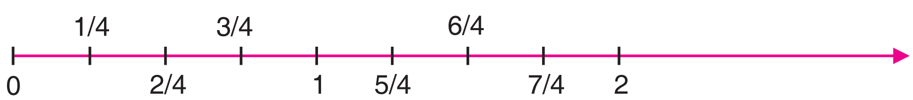
Whole numbers are
0, 1, 2, 3, 4, 5..........

(iii) Collection of natural numbers, their opposites along with zero is called the **system of integers**.



Integers are
......., – 3, – 2, – 1, 0, 1, 2, 3 ....

(iv) A part of a whole is a fraction. **Fraction** is the ratio of two natural numbers, e.g. $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{5}{4}, \frac{6}{4}, \ldots$



Properties of fractions

(a) If $\frac{p}{q}$ is a fraction, then for any natural number m,

$$\boxed{\frac{p}{q} = \frac{p \times m}{q \times m}}$$

(b) If $\frac{p}{q}$ is a fraction and a natural number m is a common divisor of p and q, then

$$\boxed{\frac{p}{q} = \frac{p \div m}{q \div m}}$$

(c) Two fractions $\frac{p}{q}$ and $\frac{r}{s}$ are said to be equivalent if

$$\boxed{p \times s = q \times r}$$

(d) A fraction $\frac{p}{q}$ is said to be in its simplest or lowest form if

p and q have no common factor other than 1.

(e) Fractions can be compared as:

(i) $\frac{p}{q} < \frac{r}{s}$ if $p \times s < q \times r$

(ii) $\frac{p}{q} = \frac{r}{s}$ if $p \times s = q \times r$

(iii) $\frac{p}{q} > \frac{r}{s}$ if $p \times s > q \times r$

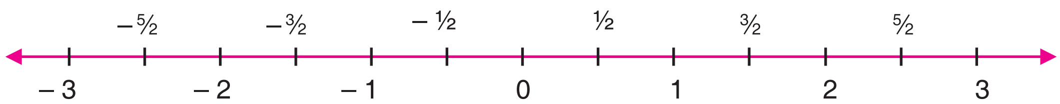
Let us do some problems to revise our memory.

Simplify the following:

1. (– 212) + 384 – (– 137)
2. (– 9) × [7 + (– 11)]
3. (– 12) × (– 10) × 6 × (– 1)
4. (– 108) ÷ (– 12)
5. (– 1331) ÷ 11
6. – 72 (– 15 – 37 – 18)

RATIONAL NUMBERS

In Class–VI, we have dealt with negative integers. In the same way, we shall be introducing negative fractions, e.g. corresponding to 1/2 we have negative fraction – 1/2.



Fractions with corresponding negative fractions and zero constitute the **system of rational numbers.**

The word 'rational' comes from the word 'ratio'.

Any number which can be expressed in the form of p/q, where p and q are integers and q ≠ 0 is known as a **Rational Number**.

See the following rational numbers.

$$\frac{1}{5} \qquad \frac{5}{-2} \qquad \frac{2}{3} \qquad \frac{-1}{5} \qquad \frac{-2}{3} \qquad \frac{-2}{-3} \qquad \frac{1}{-4}$$

Positive Rational Numbers

$$\frac{1}{5}, \frac{2}{3}, \frac{-2}{-3}$$

The rational numbers are said to be **positive** if signs of numerator and denominator are the same.

Negative Rational Numbers

$$\frac{5}{-2}, \frac{-1}{5}, \frac{-2}{3}, \frac{1}{-4}$$

The rational numbers are said to be **negative** if signs of numerator and denominator are not the same.

Remember

- Every fraction is a rational number, but every rational number need not be a fraction, e.g. $\frac{-4}{7}$, $\frac{0}{3}$ are not fractions as fractions are part of a whole which are always positive.
- All the integers are rational numbers. Integers – 50, 15, 0 can be written as, $\frac{-50}{1}$, $\frac{15}{1}$, $\frac{0}{1}$ respectively.

Worksheet 1

1. Which of the following are rational numbers?

(i) -3 (ii) $-\frac{2}{3}$ (iii) $\frac{4}{0}$ (iv) $\frac{0}{-5}$

2. Write down the rational numbers in the form $\frac{p}{q}$ whose numerators and denominators are given below:

(i) $(-5) \times 4$ and $-5 + 4$ (ii) $64 \div 4$ and $32 - 18$

3. **Which of the following are positive rational numbers?**

(i) $\frac{-2}{9}$ (ii) $\frac{3}{-5}$ (iii) $\frac{4}{9}$ (iv) $\frac{-3}{-19}$ (v) $\frac{0}{-3}$

4. **Answer the following:**

(i) Which integer is neither positive nor negative?

(ii) A rational number can always be written as $\frac{p}{q}$. Is it necessary that any number written as $\frac{q}{p}$ is a rational number?

5. **State whether the following statements are true. If not, justify your answer with an example.**

(i) Every whole number is a natural number.

(ii) Every natural number is an integer.

(iii) Every integer is a whole number.

(iv) Every integer is a rational number.

(v) Every rational number is a fraction.

(vi) Every fraction is a rational number.

PROPERTIES OF RATIONAL NUMBERS

Property I. Two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ are said to be **equivalent** if

$$p \times s = r \times q.$$

To explain the property, let us take few examples.

Example 1: Show that $\frac{4}{-7}$ and $\frac{8}{-14}$ are equivalent rational numbers.

Solution: $4 \times (-14) = -56 = 8 \times (-7)$.

Hence, $\frac{4}{-7}$ and $\frac{8}{-14}$ are equivalent rational numbers.

Example 2: Show that $\frac{5}{8}$ and $\frac{-15}{24}$ are not equivalent rational numbers.

Solution: $5 \times 24 = 120$ and $8 \times (-15) = -120$.

Hence, $5 \times 24 \neq 8 \times (-15)$.

Therefore, the given rational numbers are not equivalent.

Property II. If $\frac{p}{q}$ is a rational number and m be any integer different from zero, then

$$\frac{p}{q} = \frac{p \times m}{q \times m}.$$

Example 3: Write three rational numbers which are equivalent to $\frac{3}{5}$.

Solution: To find equivalent rational numbers, multiply numerator and denominator by any same non-zero integer.

$$\frac{3 \times 2}{5 \times 2} = \frac{6}{10} \quad \text{(Multiply numerator and denominator by 2)}$$

$$\frac{3 \times (-3)}{5 \times (-3)} = \frac{-9}{-15} \quad \text{(Multiply numerator and denominator by } -3\text{)}$$

$$\frac{3 \times 5}{5 \times 5} = \frac{15}{25} \quad \text{(Multiply numerator and denominator by 5)}$$

Hence, $\frac{6}{10}, \frac{-9}{-15}$ and $\frac{15}{25}$ are three rational numbers equivalent to $\frac{3}{5}$.

Example 4: Express $\frac{-4}{7}$ as a rational number with (i) numerator 12 (ii) denominator 28.

Solution: (i) To get numerator 12, we must multiply – 4 by – 3.

Hence, $\frac{(-4) \times (-3)}{7 \times (-3)} = \frac{12}{-21}$

Therefore, the required rational number is $\frac{12}{-21}$.

(ii) To get denominator 28, we must multiply the given denominator 7 by 4.

i.e. $\frac{(-4) \times 4}{7 \times 4} = \frac{-16}{28}$

Hence, the required rational number is $\frac{-16}{28}$.

Property III. If $\frac{p}{q}$ is a rational number and m is a common divisor of p and q then

$$\frac{p}{q} = \frac{p \div m}{q \div m}$$

Example 5: Express $\frac{-21}{49}$ as a rational number with denominator 7.

Solution: To get denominator 7, we must divide 49 by 7.

Therefore, $\frac{-21 \div 7}{49 \div 7} = \frac{-3}{7}$.

Hence, $\frac{-3}{7}$ is the required rational number.

Worksheet 2

1. In each of the following cases, show that the rational numbers are equivalent.

(i) $\frac{4}{9}$ and $\frac{44}{99}$ (ii) $\frac{7}{-3}$ and $\frac{35}{-15}$ (iii) $\frac{-3}{5}$ and $\frac{-12}{20}$

2. In each of the following cases, show that rational numbers are not equivalent.

(i) $\frac{4}{9}$ and $\frac{16}{27}$ (ii) $\frac{-100}{3}$ and $\frac{300}{9}$ (iii) $\frac{3}{-17}$ and $\frac{8}{-51}$

3. Write three rational numbers, equivalent to each of the following:

(i) $\frac{4}{7}$ (ii) $\frac{36}{108}$ (iii) $\frac{-5}{-7}$ (iv) $\frac{-72}{180}$

4. Express $\frac{3}{5}$ as rational number with numerator,

(i) -21 (ii) 150

5. Express $\frac{4}{-7}$ as a rational number with denominator,

(i) 84 (ii) -28

6. Express $\frac{90}{216}$ as a rational number with numerator 5.

7. Express $\frac{-64}{256}$ as a rational number with denominator 8.

8. Find equivalent forms of the rational numbers having a common denominator in each of the following collections of rational numbers.

(i) $\frac{2}{5}, \frac{6}{13}$ (ii) $\frac{1}{7}, \frac{2}{8}, \frac{3}{14}$ (iii) $\frac{5}{12}, \frac{7}{4}, \frac{9}{60}, \frac{11}{3}$

STANDARD FORM OF A RATIONAL NUMBER

Let us try to express a rational number in the simplest form with positive denominator.

Example 6: Express $\frac{16}{-24}$ in the simplest form with its denominator as positive.

Solution: **Step 1.** Convert denominator into positive by multiplying numerator and denominator by –1.

$$\frac{(16)\times(-1)}{(-24)\times(-1)} = \frac{-16}{24}$$

Step 2. Find HCF of 16 and 24, which is 8 in this case, and divide numerator and denominator by it.

$$\frac{-16\div 8}{24\div 8} = \frac{-2}{3}$$

The example given above explains that every rational number $\frac{p}{q}$ can be put in the simplest form with positive denominator. This form of the rational number is called its **standard form**. For this, we take the following steps.

Step 1. Make the denominator positive.

Step 2. Find the HCF m of p and q. If m = 1, then $\frac{p}{q}$ is the required form.

Step 3. If $m \neq 1$, then divide both the numerator and the denominator by m. The rational number $\frac{p\div m}{q\div m}$ so obtained is the required standard form.

> ***Note:***
>
> The numbers $\frac{-p}{q}$ and $\frac{p}{-q}$ represent the same rational number.

> **A rational number $\frac{p}{q}$ is said to be in the standard form if q is positive and the integers 'p' and 'q' have their highest common factor as 1.**

Example 7: Express $\frac{-22}{-55}$ in the standard form.

Solution: **Step 1.** $\frac{-22\times(-1)}{-55\times(-1)} = \frac{22}{55}$

Step 2. HCF of 22 and 55 is 11.

$\frac{22 \div 11}{55 \div 11} = \frac{2}{5}$ which is the standard form.

Example 8: Find x such that the rational numbers in each of the following pairs are equivalent.

(i) $\frac{x}{12}, \frac{5}{6}$ (ii) $\frac{15}{x}, \frac{-3}{8}$

Solution: (i) $\frac{x}{12}, \frac{5}{6}$ will be equivalent if

$$6 \times x = 5 \times 12$$

$$x = \frac{5 \times \cancel{12}^{2}}{\cancel{6}} = 5 \times 2 = 10$$

Hence, $x = 10$.

(ii) $\frac{15}{x}, \frac{-3}{8}$ will be equivalent if

$$15 \times 8 = (-3) \times x$$

$$x = \frac{15 \times 8}{-3} = -5 \times 8 = -40$$

Hence, $x = -40$.

Example 9: Fill in the blanks: $\frac{-3}{5} = \frac{6}{\_\_\_} = \frac{\_\_\_}{-15}$

Solution: In the first two given rational numbers, we have to find the number which when multiplied by – 3 gives the product 6. Here, the number shall be 6 ÷ (– 3) = – 2. Now, we multiply both numerator and denominator of the given rational number by – 2.

We get $\frac{-3}{5} = \frac{(-3)\times(-2)}{5\times(-2)} = \frac{6}{-10}$

To get denominator – 15,

$$\frac{-3}{5} = \frac{(-3)\times(-3)}{5\times(-3)} = \frac{9}{-15}$$ (Multiply numerator and denominator by – 3)

Thus, $\frac{-3}{5} = \frac{6}{-10} = \frac{9}{-15}$

Worksheet 3

1. Write the following rational numbers in standard form.

(i) $\frac{33}{77}$ (ii) $\frac{64}{-20}$ (iii) $\frac{-27}{-15}$ (iv) $\frac{-105}{98}$

2. Find x such that the rational numbers in each of the following pairs, become equivalent.

(i) $\frac{9}{-5}, \frac{x}{10}$ (ii) $\frac{8}{7}, \frac{x}{-35}$ (iii) $\frac{36}{x}, 2$ (iv) $\frac{x}{6}, -13$

3. Check whether the following rational numbers are in standard form. If not, write them in standard form.

(i) $\frac{-3}{19}$ (ii) $\frac{4}{-7}$ (iii) $\frac{14}{35}$ (iv) $\frac{8}{-72}$

4. Fill in the blanks.

(i) $\frac{2}{7} = \frac{8}{\_\_\_} = \frac{\_\_\_}{-63}$ (ii) $\frac{36}{\_\_\_} = \frac{-4}{9} = \frac{-84}{\_\_\_}$ (iii) $\frac{105}{\_\_\_} = \frac{\_\_\_}{-99} = \frac{-5}{-11}$

ABSOLUTE VALUE OF A RATIONAL NUMBER

We have studied in Class–VI that absolute value of an integer is its numerical value without taking the sign into account, e.g. $|-3| = 3$, $|3| = 3$, $|0| = 0$

The absolute value of a rational number is written in the following ways.

Absolute value of $\frac{4}{5}$ is $\left|\frac{4}{5}\right| = \frac{4}{5}$

Absolute value of $\frac{-4}{5}$ is $\left|\frac{-4}{5}\right| = \frac{4}{5}$

Absolute value of $\frac{4}{-5}$ is $\left|\frac{4}{-5}\right| = \frac{4}{5}$

Absolute value of $\frac{-4}{-5}$ is $\left|\frac{-4}{-5}\right| = \frac{4}{5}$

Remember

- Absolute value of every rational number other than zero is positive.
- The absolute value of zero is zero itself.
- Absolute value of a rational number is greater than or equal to the number itself.

Worksheet 4

1. Find the absolute value of the following rational numbers.

(i) $\frac{1}{-5}$ (ii) $\frac{7}{9}$ (iii) $\frac{0}{-4}$ (iv) $\frac{-3}{-2}$

2. Compare the absolute values of the rational numbers in the following pairs.

(i) $\frac{3}{-7}, \frac{-3}{7}$ (ii) $\frac{-5}{7}, \frac{4}{3}$ (iii) $\frac{-4}{5}, -3$

3. Find all the rational numbers whose absolute value is–

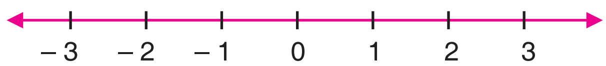
(i) $\frac{2}{5}$ (ii) 0 (iii) $\frac{3}{4}$

REPRESENTATION OF RATIONAL NUMBERS ON A NUMBER LINE

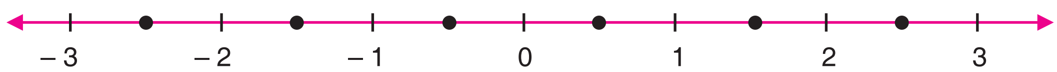
You have dealt with the definition and properties of rational numbers. Now, you will learn how to plot rational numbers on a number line.

Let us mark the rational numbers $\frac{-5}{2}, \frac{-3}{2}, \frac{-1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ on the number line.

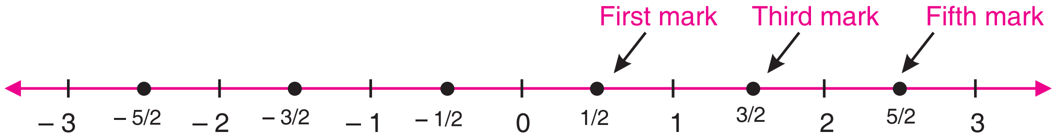
Step 1. Mark integers on the number line.



Step 2. Divide each unit segment into two equal parts (equal to denominator).



Step 3. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ are represented by first, third and fifth mark respectively lying to the right of zero.



$\frac{-1}{2}, \frac{-3}{2}, \frac{-5}{2}$ are represented by first, third and fifth mark respectively lying to the left of zero.

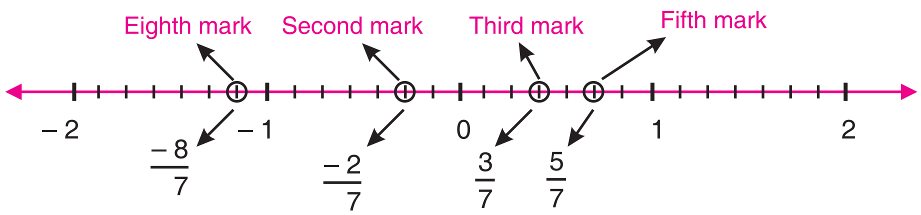
Example 10: Represent $\frac{3}{7}, \frac{5}{7}, \frac{-8}{7}, \frac{-2}{7}$ on number line.

Solution:

Step 1. Mark integers on number line.

Step 2. Divide each unit segment into seven equal parts.

Step 3. Third and fifth mark on right side of zero represent $\frac{3}{7}$ and $\frac{5}{7}$. Eighth and second mark on left side of zero represent $\frac{-8}{7}$ and $\frac{-2}{7}$ respectively.



Worksheet 5

1. State whether the following statements are true. If not, justify your answer.

(i) On a number line, all the numbers to the right of zero are positive.

(ii) Rational number $\frac{-7}{-19}$ lies to the left of zero on the number line.

(iii) On a number line, numbers become progressively larger as we move away from zero.

(iv) Rational numbers $\frac{2}{3}$ and $\frac{-2}{3}$ are at equal distance from zero.

(v) On a number line, number lying left to a given number is greater.

2. Mark the following rational numbers on number line.

(i) $\frac{4}{5}$ (ii) $\frac{-8}{3}$ (iii) $\frac{5}{2}$ (iv) $\frac{-7}{3}$

3. Represent the following rational numbers on a number line.

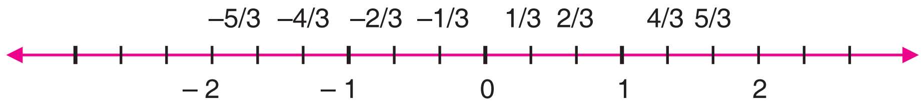
(i) $\frac{-3}{5}$ (ii) $\frac{2}{-3}$ (iii) $\frac{3}{4}$ (iv) $\frac{-4}{-7}$

COMPARING RATIONAL NUMBERS

Rational numbers can be compared in two different ways.

I. BY REPRESENTING ON NUMBER LINE

Rational numbers can be compared easily when they are represented on the number line.



Any number on the number line is greater than any other number lying to the left of it.

Any number on a number line is less than any other number lying to the right of it.

Therefore, from the above number line it is clear that

$$\frac{2}{3} < \frac{5}{3}, \quad 1 < \frac{4}{3}, \quad \frac{-5}{3} < \frac{-1}{3}, \quad \frac{-4}{3} < \frac{1}{3}, \quad \text{etc.}$$

and $$\frac{4}{3} > \frac{1}{3}, \frac{4}{3} > \frac{-2}{3}, \quad \frac{-1}{3} > \frac{-4}{3}, \text{ etc.}$$

II. WITHOUT REPRESENTING ON NUMBER LINE

Without representing the rational numbers on the number line, we can compare them by the method similar to the one used for fractional numbers.

If two rational numbers have the same positive denominator, the number with the larger numerator will be greater than the one with smaller numerator.

Example 11: Compare,

(i) $\frac{2}{7}$ and $\frac{5}{7}$ (ii) $\frac{-6}{17}$ and $\frac{-13}{17}$

Solution: (i) The rational numbers $\frac{2}{7}$ and $\frac{5}{7}$ have same denominator, therefore, smaller the numerator, smaller will be the rational number. Since 2 < 5, therefore, $\frac{2}{7} < \frac{5}{7}$.

(ii) $\frac{-6}{17}$ and $\frac{-13}{17}$ have the same denominator. Therefore, we shall compare the numerators–

$$-6 > -13$$

Therefore, $\frac{-6}{17} > \frac{-13}{17}$.

> **If two rational numbers have different denominators, then first make denominators equal and then compare.**

Example 12: Compare $\frac{7}{5}$ and $\frac{8}{7}$.

Solution: First, convert the rational numbers to have the same positive denominator.

$$\left.\begin{aligned}\frac{7}{5} &= \frac{7\times 7}{5\times 7} = \frac{49}{35}\\ \frac{8}{7} &= \frac{8\times 5}{7\times 5} = \frac{40}{35}\end{aligned}\right] \quad \text{(Denominators are same)}$$

Now, compare $\frac{49}{35}$ and $\frac{40}{35}$

As $49 > 40$, therefore, $\frac{49}{35} > \frac{40}{35}$

Hence, $\frac{7}{5} > \frac{8}{7}$.

Example 13: Compare the rational numbers $\frac{-4}{9}$ and $\frac{5}{-6}$.

Solution: First write $\frac{5}{-6}$ in standard form, i.e. $\frac{-5}{6}$.

Now, convert them to have the same denominator.

$$\frac{-4}{9} \times \frac{2}{2} = \frac{-8}{18}$$

$$\frac{-5}{6} \times \frac{3}{3} = \frac{-15}{18}$$

Now, compare $\quad \frac{-8}{18}$ and $\frac{-15}{18}$

Since, numerator $-8 > -15$, therefore, $\frac{-8}{18} > \frac{-15}{18}$.

Hence, $\quad \frac{-4}{9} > \frac{5}{-6}$.

There is yet another method to compare two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ with unequal denominators. It is assumed that q and s are both positive integers.

To compare $\frac{p}{q}$ and $\frac{r}{s}$, we may compare ps and qr $\left(\frac{p}{q} \times \frac{r}{s}\right)$

Find products ps and qr.

If $\quad ps > qr \quad$ then $\quad \frac{p}{q} > \frac{r}{s}$.

If $\quad ps < qr \quad$ then $\quad \frac{p}{q} < \frac{r}{s}$.

Example 14: Compare $\frac{5}{3}$ and $\frac{2}{7}$.

Solution: $\quad \frac{5}{3} \times \frac{2}{7}$

The products are $5 \times 7 = 35$ and $3 \times 2 = 6$

Since, $35 > 6$, therefore, $\frac{5}{3} > \frac{2}{7}$.

Example 15: Compare $\frac{-5}{7}$ and $\frac{4}{-9}$.

Solution: First write $\frac{4}{-9}$ in standard form as $\frac{-4}{9}$.

Now, find the products $\frac{-5}{7} \times \frac{-4}{9}$

The products are $-5 \times 9 = -45$ and $7 \times (-4) = -28$

Since, $-45 < -28$, therefore, $\frac{-5}{7} < \frac{-4}{9}$.

Worksheet 6

1. Determine which rational number is greater in each case.

(i) $\frac{5}{8}, \frac{-3}{7}$

(ii) $\frac{2}{3}, \frac{8}{9}$

(iii) $\frac{-4}{3}, \frac{-6}{7}$

(iv) $\frac{-8}{3}, \frac{19}{-6}$

(v) $\frac{-3}{-13}, \frac{-5}{-21}$

(vi) $\frac{-7}{11}, \frac{5}{-8}$

2. Find the value of x, if–

(i) $\frac{3}{-5} = \frac{x}{15}$

(ii) $\frac{9}{15} = \frac{x}{-50}$

(iii) $\frac{36}{x} = -4$

(iv) $\frac{7}{-x} = 7$

3. Compare the rational numbers.

(i) $\frac{-2}{9}, \frac{8}{-36}$

(ii) $\frac{5}{9}, \frac{4}{6}$

(iii) $\frac{-7}{-8}, \frac{14}{17}$

(iv) $\frac{-4}{7}, \frac{5}{-9}$

(v) $\frac{-5}{8}, \frac{-3}{4}$

(vi) $\frac{6}{7}, \frac{-54}{-63}$

4. Arrange the following in ascending order.

(i) $\frac{4}{7}, \frac{5}{9}, \frac{2}{5}$

(ii) $\frac{-3}{4}, \frac{-5}{-12}, \frac{-7}{16}$

5. Arrange the following in descending order.

(i) $\frac{2}{5}, \frac{-1}{2}, \frac{8}{-15}, \frac{-3}{-10}$

(ii) $\frac{-7}{10}, \frac{8}{-15}, \frac{19}{30}, \frac{-2}{-5}$

VALUE BASED QUESTIONS

1. **Sukhdev, a farmer, had a son and a daughter. He decided to divide his property among his children. He gave $\frac{2}{5}$ of the property to his son and $\frac{4}{10}$ to his daughter, and rest to a charitable trust.**
   (a) Whose share was more, son's or daughter's?
   (b) What do you feel about Sukhdev's decision? Which values are exhibited here?

2. **Kavita along with her family was planning a vacation at a hill station. But, they were confused where to go. Kavita's mother asked her to find out the maximum temperature of few hill stations for deciding on the place to visit. She checked the weather report on the internet and found that—**

   Simla's temperature = $\left(\frac{-7}{2}\right)^{\circ}$ C

   Dalhousie's temperature = – 5°C

   Manali's temperature = $\left(\frac{-8}{5}\right)^{\circ}$ C

   (a) Arrange the temperatures of these hill stations in ascending order.
   (b) Which place will they decide to visit?
   (c) What value is exhibited in the above situation?

BRAIN TEASERS

1. **A. Tick (✓) the correct option.**

   (a) The value of x such that $\frac{-3}{8}$ and $\frac{x}{-24}$ are equivalent rational numbers is–

   (i) 64 (ii) – 64

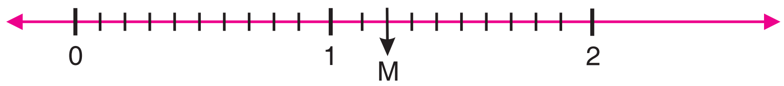
   (iii) – 9 (iv) 9

   (b) Which of the following is a negative rational number?

   (i) $\frac{-15}{-4}$ (ii) 0

   (iii) $\frac{-5}{7}$ (iv) $\frac{4}{9}$

(c) In the given number line, which of the following rational numbers does the point M represent?



(i) $\frac{2}{8}$ (ii) $\frac{6}{5}$ (iii) $\frac{2}{3}$ (iv) $\frac{12}{5}$

(d) Which is the greatest rational number out of $\frac{5}{-11}, \frac{-5}{12}, \frac{5}{-17}$?

(i) $\frac{5}{-11}$ (ii) $\frac{-5}{12}$

(iii) $\frac{5}{-17}$ (iv) cannot be compared

(e) Which of the following rational numbers is the smallest?

(i) $\left|\frac{7}{11}\right|$ (ii) $\left|\frac{-8}{11}\right|$ (iii) $\left|\frac{-2}{11}\right|$ (iv) $\left|\frac{-9}{-11}\right|$

B. Answer the following questions.

(a) Find the average of the rational numbers $\frac{4}{5}, \frac{2}{3}, \frac{5}{6}$.

(b) How will you write $\frac{12}{-18}$ in the standard form?

(c) How many rational numbers are there between any two rational numbers?

(d) On the number line, the rational number $\frac{-5}{-7}$ lies on which side of zero?

(e) Express $\frac{-7}{-8}$ as a rational number with denominator 40.

2. State whether the following statements are true. If not, then give an example in support of your answer.

(i) If $\frac{p}{q} > \frac{r}{s}$ then $\left|\frac{p}{q}\right| > \left|\frac{r}{s}\right|$

(ii) If $|x| = |y|$ then $x = y$

(iii) $\frac{p}{q}$ is a non-zero rational number in standard form. It is necessary that rational number $\frac{q}{p}$ will also be in standard form.

3. **Represent $5\frac{1}{3}$ and $\frac{-29}{4}$ on a number line.**

4. **Arrange the following rational numbers in descending order.**

   $\frac{-3}{10}, \frac{-7}{-5}, \frac{9}{-15}, \frac{18}{30}$

5. **On a number line, what is the length of the line-segment joining,**

   (i) 3 and – 3?
   (ii) $\frac{1}{2}$ and $\frac{-1}{2}$?
   (iii) $\frac{1}{2}$ and $2\frac{1}{2}$?
   (iv) $\frac{-1}{2}$ and $-2\frac{1}{2}$?

6. **Find the values of x in each of the following:**

   (i) $\frac{23}{x} = \frac{2}{-8}$
   (ii) $\frac{x}{9} = \frac{19}{3}$
   (iii) $\frac{15}{-x} = \frac{1}{-7}$

7. **Compare the numbers in each of following pairs of numbers.**

   (i) $\frac{-5}{7}, \frac{9}{-13}$
   (ii) $\frac{-4}{9}, \frac{-3}{7}$
   (iii) $\frac{-3}{-5}, \frac{12}{20}$
   (iv) $\left|\frac{-4}{5}\right|, \left|\frac{-5}{4}\right|$
   (v) $\left|\frac{5}{7}\right|, \left|\frac{-15}{21}\right|$
   (vi) $\left|\frac{-8}{-9}\right|, \left|\frac{-3}{9}\right|$

8. **Fill in the following blank squares.**

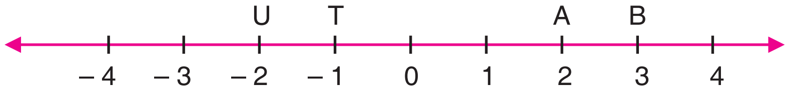
   (i) $\frac{3}{5} = \frac{138}{\square}$
   (ii) $\frac{7}{9} = \frac{\square}{108}$
   (iii) $\frac{\square}{-15} = \frac{48}{90}$
   (iv) $\frac{121}{\square} = \frac{-11}{12}$

HOTS

The points P, Q, R, S, T, U, A and B are on the number line representing integers such that—

TR = RS = SU and AP = PQ = QB

Locate and write the rational numbers represented by points P, Q, R, and S.



YOU MUST KNOW

1. A number of the form $\frac{p}{q}$ is called a fraction, if p and q are natural numbers. If p and q are integers and $q \neq 0$, then it is said to be a rational number.
2. Every integer and fraction is a rational number but the converse may not be true.
3. A rational number is said to be positive if both numerator and denominator are of same sign. If numerator and denominator are of opposite signs, then rational number is said to be negative.
4. If $\frac{p}{q}$ be a rational number and m be any integer different from zero, then $\frac{p}{q} = \frac{p \times m}{q \times m}$.
5. If $\frac{p}{q}$ be a rational number and m be a common divisor of p and q, then $\frac{p}{q} = \frac{p \div m}{q \div m}$.
6. A rational number $\frac{p}{q}$ is said to be in standard form if q is positive and HCF of p and q is 1.
7. Two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ are said to be equivalent (equal) if $p \times s = q \times r$.
8. Every rational number can be represented on the number line.
9. If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers with q and s positive integers then $\frac{p}{q} > \frac{r}{s}$ if $p \times s > q \times r$,

   $\frac{p}{q} < \frac{r}{s}$ if $p \times s < q \times r$ and $\frac{p}{q} = \frac{r}{s} =$ if $p \times s = q \times r$.
10. Every rational number has an absolute value which is greater than or equal to zero.

CHAPTER 2

OPERATIONS ON RATIONAL NUMBERS

In Class-V and VI, we have dealt with the operations (addition, subtraction, multiplication, division) on fractions and integers. Now, we shall study these operations and their properties in case of Rational Numbers.

ADDITION OF RATIONAL NUMBERS

I. When the rational numbers have the same denominator.

Example 1: Add $\frac{3}{7}$ and $\frac{6}{7}$.

Solution: $\frac{3}{7}+\frac{6}{7} = \frac{3+6}{7}$ ← Addition of numerators; ← Common denominator

$= \frac{9}{7}$

Add numerators of the rational numbers and then divide by the common denominator.

Example 2: Find the sum of (i) $\frac{3}{10}, \frac{-7}{10}$ (ii) $\frac{9}{-11}, \frac{8}{-11}$

Solution: (i) $\frac{3}{10}+\frac{-7}{10}=\frac{3+(-7)}{10}=\frac{3-7}{10}=\frac{-4}{10}=\frac{-2}{5}$ (standard form)

(ii) $\frac{9}{-11}+\frac{8}{-11}=\frac{9+8}{-11}=\frac{17}{-11}=\frac{-17}{11}$ (standard form)

II. When the rational numbers have different denominators.

Example 3: Add $\frac{1}{4}$ and $\frac{2}{3}$.

Solution: $\frac{1}{4}+\frac{2}{3}$

LCM of denominators 4 and 3 is 12.

$$\frac{1}{4} = \frac{1\times 3}{4\times 3} = \frac{3}{12}$$

$$\frac{2}{3} = \frac{2\times 4}{3\times 4} = \frac{8}{12}$$

denominators are same.

Add $\frac{3}{12}+\frac{8}{12} = \frac{3+8}{12} = \frac{11}{12}$

> Take the LCM of denominators and then add the rational numbers.

Example 4: Find the sum of (i) $\frac{-3}{11}, \frac{-2}{7}$ (ii) $\frac{7}{-16}, \frac{-3}{4}$

Solution: (i) $\frac{-3}{11}+\frac{(-2)}{7}$

LCM of denominators 11 and 7 is 77.

$$\frac{-3}{11} = \frac{-3\times 7}{11\times 7} = \frac{-21}{77}$$

$$\frac{-2}{7} = \frac{-2\times 11}{7\times 11} = \frac{-22}{77}$$

Now add $\frac{-21}{77}+\frac{-22}{77} = \frac{(-21)+(-22)}{77}$

$$= \frac{-21-22}{77} = \frac{-43}{77}.$$

(ii) $\frac{7}{(-16)}+\frac{(-3)}{4}$

$$\frac{7}{-16} = \frac{7\times(-1)}{-16\times(-1)} = \frac{-7}{16}$$ (standard form)

Now, we have to add, $\frac{-7}{16}$ and $\frac{-3}{4}$.

LCM of denominators 16 and 4 is 16.

$$\frac{-7}{16}+\frac{-3}{4} = \frac{-7}{16}+\frac{-3\times 4}{4\times 4}$$

$$= \frac{-7}{16}+\frac{(-12)}{16}$$

$$= \frac{-7-12}{16} = \frac{-19}{16}.$$

Note:

From above examples, it is clear that the sum of any two rational numbers is also a rational number.

PROPERTIES OF ADDITION OF RATIONAL NUMBERS

Like other numbers (natural numbers, whole numbers, fractions and integers), addition of rational numbers also satisfy the following properties.

1. Verify that the sum of two rational numbers, $\frac{-2}{7}$ and $\frac{1}{4}$ remains the same even if the order of addends is changed.

First, find the sum $\frac{-2}{7} + \frac{1}{4}$

$$\frac{-2 \times 4}{7 \times 4} + \frac{1 \times 7}{4 \times 7} \quad \text{(LCM of 7 and 4 is 28)}$$

$$\frac{-8}{28} + \frac{7}{28} = \frac{-8+7}{28} = \frac{-1}{28}$$

Now, change the order and find the sum.

$$\frac{1}{4} + \frac{-2}{7} = \frac{1 \times 7}{4 \times 7} + \frac{-2 \times 4}{7 \times 4} = \frac{7}{28} + \frac{(-8)}{28} = \frac{7+(-8)}{28} = \frac{-1}{28}$$

What do you observe?

Property 1: **The sum remains the same even if we change the order of addends, i.e. for two rational numbers x and y, x + y = y + x.**
This is commutative law of addition.

2. Verify that the sum of three rational numbers $\frac{1}{2}, \frac{2}{3}$ and $\frac{5}{4}$ remains the same even after changing the grouping.

$$\left(\frac{1}{2} + \frac{2}{3}\right) + \frac{5}{4} \leftarrow \left(\text{First add } \frac{1}{2} \text{ and } \frac{2}{3}\right)$$

$$= \left(\frac{1 \times 3}{2 \times 3} + \frac{2 \times 2}{3 \times 2}\right) + \frac{5}{4}$$

$$\frac{1}{2} + \left(\frac{2}{3} + \frac{5}{4}\right) \leftarrow \text{(Grouping is changed)}$$

$$= \frac{1}{2} + \left(\frac{2 \times 4}{3 \times 4} + \frac{5 \times 3}{4 \times 3}\right)$$

$$= \left(\frac{3}{6} + \frac{4}{6}\right) + \frac{5}{4}$$

$$= \left(\frac{3+4}{6}\right) + \frac{5}{4}$$

$$= \frac{7}{6} + \frac{5}{4}$$

$$= \frac{7\times 2}{6\times 2} + \frac{5\times 3}{4\times 3}$$

$$= \frac{14}{12} + \frac{15}{12}$$

$$= \frac{14+15}{12}$$

$$= \frac{29}{12}$$

$$= \frac{1}{2} + \left(\frac{8}{12} + \frac{15}{12}\right)$$

$$= \frac{1}{2} + \left(\frac{8+15}{12}\right)$$

$$= \frac{1}{2} + \frac{23}{12}$$

$$= \frac{1\times 6}{2\times 6} + \frac{23}{12}$$

$$= \frac{6}{12} + \frac{23}{12}$$

$$= \frac{6+23}{12}$$

$$= \frac{29}{12}$$

Same

Property 2: **Sum of three rational numbers remains same even after changing the grouping of the addends, i.e. if x, y and z are three rational numbers, then $(x + y) + z = x + (y + z)$**
This is known as associative law of addition.

3. Find the sum of $\frac{5}{3}$ and 0.

$$\frac{5}{3} + 0 = \frac{5}{3} + \frac{0}{3} = \frac{5+0}{3} = \frac{5}{3}$$

Similarly, $0 + \frac{(-27)}{23} = \frac{0}{23} + \frac{(-27)}{23} = \frac{0-27}{23} = \frac{-27}{23}$

Property 3: **When zero is added to any rational number, the sum is the rational number itself, i.e. if x is a rational number, then $0 + x = x + 0 = x$.**
Zero is called the identity element of addition.

4. Find the sum of $\frac{7}{9}$ and $\frac{-7}{9}$.

$$\frac{7}{9} + \frac{-7}{9} = \frac{7+(-7)}{9} = \frac{0}{9} = 0$$

Zero is the identity element of addition.

Similarly, $\frac{-2}{3} + \frac{2}{3} = \frac{(-2)+2}{3} = \frac{0}{3} = 0$

Property 4: **Every rational number has an additive inverse such that their sum is equal to zero. If x is a rational number, then – x is a rational number such that x + (– x) = 0.**
– x is called additive inverse of x. It is also called the negative of x.

Following are a few examples to illustrate these properties.

Example 5: Simplify, $\frac{3}{7}+\frac{5}{7}+\frac{-3}{7}+\frac{-5}{7}$.

Solution: To find the sum of three or more rational numbers, we may arrange them in any order we like. The arrangement does not alter the sum.

First method

$$\frac{3}{7}+\frac{5}{7}+\frac{-3}{7}+\frac{-5}{7}$$

$$=\left(\frac{3}{7}+\frac{-3}{7}\right)+\left(\frac{5}{7}+\frac{-5}{7}\right)$$

$$= 0+0=0$$

Sum of a number and its additive inverse is zero.

Second method

$$\left(\frac{3}{7}+\frac{5}{7}\right)+\left(\frac{-3}{7}+\frac{-5}{7}\right)$$

$$=\left(\frac{3+5}{7}\right)+\left(\frac{-3+(-5)}{7}\right)$$

$$=\frac{8}{7}+\left(\frac{-3-5}{7}\right)$$

$$=\frac{8}{7}+\left(\frac{-8}{7}\right)=0$$

Observe that the first method is simpler than the second.

Example 6: Find the value of $\frac{3}{5}+\frac{5}{4}+\frac{-7}{15}+\frac{-3}{8}$

Solution:

$$\frac{3}{5}+\frac{5}{4}+\frac{-7}{15}+\frac{-3}{8}$$

$$=\left(\frac{3}{5}+\frac{-7}{15}\right)+\left(\frac{5}{4}+\frac{-3}{8}\right)$$

(This grouping simplifies the calculation.)

$$=\left(\frac{3\times 3}{5\times 3}+\frac{-7}{15}\right)+\left(\frac{5\times 2}{4\times 2}+\frac{-3}{8}\right)=\left(\frac{9}{15}+\frac{-7}{15}\right)+\left(\frac{10}{8}+\frac{-3}{8}\right)$$

$$=\frac{9+(-7)}{15}+\frac{10+(-3)}{8}=\frac{9-7}{15}+\frac{10-3}{8}=\frac{2}{15}+\frac{7}{8}$$

$$=\frac{2\times 8}{15\times 8}+\frac{7\times 15}{8\times 15}=\frac{16}{120}+\frac{105}{120}=\frac{16+105}{120}=\frac{121}{120}$$

Now, if we change the grouping.

$$\left(\frac{3}{5}+\frac{5}{4}\right)+\left(\frac{-7}{15}+\frac{-3}{8}\right)$$

$$=\left(\frac{3\times 4}{5\times 4}+\frac{5\times 5}{4\times 5}\right)+\left(\frac{-7\times 8}{15\times 8}+\frac{-3\times 15}{8\times 15}\right)$$

$$=\left(\frac{12}{20}+\frac{25}{20}\right)+\left(\frac{-56}{120}+\frac{-45}{120}\right)$$

$$=\left(\frac{12+25}{20}\right)+\left(\frac{-56+(-45)}{120}\right)$$

$$=\frac{37}{20}+\left(\frac{-56-45}{120}\right)=\frac{37}{20}+\frac{(-101)}{120}$$

$$=\frac{37\times 6}{20\times 6}+\frac{(-101)}{120}=\frac{222}{120}+\frac{(-101)}{120}$$

$$=\frac{222-101}{120}=\frac{121}{120}.$$

(This grouping makes the calculations longer and slightly difficult than the first grouping.)

Worksheet 1

1. Add the following:

(i) $\frac{2}{7}+\frac{5}{7}$ (ii) $\frac{-5}{9}+\frac{7}{9}$ (iii) $2\frac{3}{11}+\frac{9}{-11}$

(iv) $\frac{-5}{4}+\frac{5}{4}$ (v) $-2\frac{5}{6}+\frac{13}{-6}$ (vi) $\frac{-21}{3}+\frac{18}{3}$

2. Find the values of:

(i) $\frac{4}{9}+\frac{7}{4}$ (ii) $\frac{-7}{11}+\frac{1}{4}$ (iii) $\frac{5}{8}+\frac{-3}{5}$

(iv) $\frac{10}{63}+\frac{6}{7}$ (v) $\frac{-7}{64}+\frac{3}{-16}$ (vi) $\frac{5}{12}+\frac{-9}{20}$

3. Verify x + y = y + x for following values of x and y.

(i) $x=\frac{5}{7}, y=\frac{-3}{2}$ (ii) $x=5, y=\frac{3}{2}$

(iii) $x=\frac{-5}{14}, y=\frac{-1}{21}$ (iv) $x=-8, y=\frac{9}{2}$

4. **Verify x + (y + z) = (x + y) + z for following values of x, y and z.**

(i) $x = \frac{3}{4}, y = \frac{5}{6}, z = \frac{-7}{8}$

(ii) $x = \frac{2}{3}, y = \frac{-5}{6}, z = \frac{-7}{9}$

(iii) $x = \frac{3}{5}, y = \frac{-6}{9}, z = \frac{2}{10}$

(iv) $x = \frac{-3}{5}, y = \frac{-7}{10}, z = \frac{-8}{15}$

5. **Simplify.**

(i) $\frac{-3}{10} + \frac{12}{-10} + \frac{14}{10}$

(ii) $\frac{-5}{10} + \frac{6}{13} + 8$

(iii) $\frac{-5}{10} + \frac{9}{7} + \frac{3}{20} + \frac{-11}{14}$

(iv) $\frac{5}{36} + \frac{-7}{8} + \frac{6}{-72} + \frac{-3}{-12}$

6. **For $x = \frac{1}{5}$ and $y = \frac{3}{7}$, verify that $-(x + y) = (-x) + (-y)$.**

7. **Write True or False for the following statements.**

(i) $\frac{-2}{-3}$ is the additive inverse of $\frac{2}{3}$.

(ii) $\frac{2}{3} + \frac{4}{5}$ is a rational number.

(iii) $\frac{-5}{3} + \frac{5}{-3}$ is equal to zero.

(iv) 1 is the identity element of addition.

(v) $\frac{-9}{7} + 0 = 0$.

(vi) Additive inverse of $\frac{-3}{5}$ is $\frac{3}{5}$.

(vii) Negative of a negative rational number is negative.

SUBTRACTION OF RATIONAL NUMBERS

Remember

Subtracting y from x is same as adding the additive inverse of y to x, i.e. $x - y = x + (-y)$

Let us explain it with the help of some illustrations.

Example 7: Subtract $\frac{7}{5}$ from $\frac{6}{3}$.

Solution: $\frac{6}{3}-\frac{7}{5}$

$$= \frac{6}{3}+\left(\frac{-7}{5}\right) \qquad \leftarrow \text{Adding } \frac{6}{3} \text{ and additive inverse of } \frac{7}{5}.$$

$$= \frac{6\times 5}{3\times 5}+\frac{(-7)\times 3}{5\times 3}$$

$$= \frac{30}{15}+\frac{(-21)}{15} = \frac{30+(-21)}{15}$$

$$= \frac{30-21}{15} = \frac{9}{15} = \frac{3}{5} \qquad \leftarrow \text{Standard form}$$

Example 8: Subtract $\frac{5}{63}$ from $\frac{-6}{7}$.

Solution: $\frac{-6}{7}-\frac{5}{63} = \frac{-6\times 9}{7\times 9}-\frac{54}{63}$

$$= \frac{-54}{63}-\frac{5}{63} = \frac{-54-5}{63} = \frac{-59}{63}.$$

Note:

From above examples, it is clear that difference of two rational numbers is a rational number.

PROPERTIES OF SUBTRACTION OF RATIONAL NUMBERS

1. **Verify that the difference of two rational numbers, $\frac{2}{3}$ and $\frac{7}{6}$ does not remain same if the order of numbers is changed.**

$\frac{2}{3}-\frac{7}{6}$	$\frac{7}{6}-\frac{2}{3}$
$= \frac{2\times 2}{3\times 2}-\frac{7}{6}$	$= \frac{7}{6}-\frac{2\times 2}{3\times 2}$

$= \frac{4}{6} - \frac{7}{6}$	$= \frac{7}{6} - \frac{4}{6}$
$= \frac{4-7}{6}$	$= \frac{7-4}{6}$
$= \frac{-3}{6}$	$= \frac{3}{6}$

Not Same

Property 1: **For rational numbers x and y, $x - y \neq y - x$ in general. In fact, $x - y = -(y - x)$, i.e. commutative property does not hold true for subtraction.**

2. Let us now observe the following cases.

(Grouping is changed)

$\left(\frac{5}{4} - \frac{3}{2}\right) - \frac{2}{3}$	$\frac{5}{4} - \left(\frac{3}{2} - \frac{2}{3}\right)$
$= \left(\frac{5}{4} - \frac{3 \times 2}{2 \times 2}\right) - \frac{2}{3}$	$= \frac{5}{4} - \left(\frac{3 \times 3}{2 \times 3} - \frac{2 \times 2}{3 \times 2}\right)$
$= \left(\frac{5}{4} - \frac{6}{4}\right) - \frac{2}{3}$	$= \frac{5}{4} - \left(\frac{9}{6} - \frac{4}{6}\right)$
$= \frac{5-6}{4} - \frac{2}{3}$	$= \frac{5}{4} - \left(\frac{9-4}{6}\right)$
$= \frac{-1}{4} - \frac{2}{3}$	$= \frac{5}{4} - \frac{5}{6}$
$= \frac{-1 \times 3}{4 \times 3} - \frac{2 \times 4}{3 \times 4}$	$= \frac{5 \times 3}{4 \times 3} - \frac{5 \times 2}{6 \times 2}$
$= \frac{-3}{12} - \frac{8}{12}$	$= \frac{15}{12} - \frac{10}{12}$
$= \frac{-3-8}{12}$	$= \frac{15-10}{12}$
$= \frac{-11}{12}$	$= \frac{5}{12}$

Not Same

Property 2: **For rational numbers x, y, z, in general, $(x - y) - z \neq x - (y - z)$, i.e. the associative property does not hold true for subtraction.**

3. For all rational numbers x, we have

$$x - 0 = x$$

but $\qquad 0 - x = -x$

Therefore,

Property 3: **Identity element for subtraction does not exist.**

Property 4: **Since the identity element for subtraction does not exist, the question for finding inverse for subtraction does not arise.**

Following are a few examples to illustrate these properties.

Example 9: What number should be added to $\frac{-3}{5}$ so as to get $\frac{3}{7}$?

Solution: The required number will be obtained by subtracting $\frac{-3}{5}$ from $\frac{3}{7}$.

Therefore, required number shall be $= \frac{3}{7} - \left(\frac{-3}{5}\right) = \frac{3}{7} + \frac{3}{5}$

$$= \frac{3 \times 5}{7 \times 5} + \frac{3 \times 7}{5 \times 7} = \frac{15}{35} + \frac{21}{35}$$

$$= \frac{15 + 21}{35} = \frac{36}{35}$$

Example 10: Simplify: (i) $\frac{5}{4} - \frac{7}{6} - \left(\frac{-2}{3}\right)$ $\qquad$ (ii) $\frac{-3}{5} - \left(\frac{-4}{15}\right) - \left(\frac{7}{-10}\right)$

Solution: (i) $\frac{5}{4} - \frac{7}{6} - \left(\frac{-2}{3}\right)$

2 has come by dividing 12 by 6

3 has come by dividing 12 by 4

4 has come by dividing 12 by 3

$$\frac{5 \times 3 - 7 \times 2 - (-2) \times 4}{12}$$ (LCM of 4, 6 and 3 is 12)

$$= \frac{15 - 14 - (-8)}{12} = \frac{15 - 14 + 8}{12}$$

$$= \frac{1 + 8}{12} = \frac{9}{12} = \frac{3}{4}$$ (standard form)

(ii) $\frac{-3}{5}-\left(\frac{-4}{15}\right)-\left(\frac{7}{-10}\right)$

First check whether all rational numbers are in standard form. Here $\frac{7}{-10}$ is not in standard form. Standard form of $\frac{7}{-10}$ is $\frac{-7}{10}$.

Now, we have $\frac{-3}{5}-\left(\frac{-4}{15}\right)-\left(\frac{-7}{10}\right)$

$$= \frac{(-3)\times 6-(-4)\times 2-(-7)\times 3}{30} \qquad \text{(LCM of 5, 10, 15 is 30)}$$

$$= \frac{-18-(-8)-(-21)}{30}$$

$$= \frac{-18+8+21}{30} = \frac{-10+21}{30} = \frac{11}{30}.$$

Worksheet 2

1. Find the value of—

(i) $\frac{6}{7}-\frac{-5}{7}$

(ii) $\frac{5}{24}-\frac{7}{36}$

(iii) $\frac{9}{10}-\frac{7}{-15}$

(iv) $\frac{-3}{8}-\frac{(-6)}{20}$

2. Subtract.

(i) $\frac{5}{9}$ from $\frac{-7}{9}$

(ii) $\frac{-5}{7}$ from 0

(iii) $\frac{5}{11}$ from $\frac{-8}{23}$

(iv) $\frac{-2}{9}$ from $\frac{7}{6}$

3. The sum of two rational numbers is – 5. If one of the number is $\frac{2}{3}$, find the other.

4. What number should be added to $\frac{-3}{7}$ so as to get 1?

5. What number should be subtracted from – 1 so as to get $\frac{5}{3}$?

6. **Simplify.**

(i) $\frac{-4}{5} - \frac{3}{15} + \frac{7}{20}$

(ii) $\frac{-5}{13} - \frac{-3}{26} - \frac{9}{-52}$

(iii) $\frac{7}{24} + \frac{5}{12} - \frac{11}{18}$

(iv) $\frac{-11}{30} - \frac{8}{15} + \frac{7}{6} + \frac{-2}{5}$

7. **Find the values of x – y and y – x for $x = \frac{2}{3}$, $y = \frac{5}{9}$. Are they equal?**

8. **For $x = \frac{1}{10}$, $y = \frac{-3}{5}$, $z = \frac{7}{20}$, find the values of the expressions (x – y) – z and x – (y – z). Are they equal?**

MULTIPLICATION OF RATIONAL NUMBERS

You have learnt in Class-V how to multiply two fractions. Let us recall.

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15} \quad \begin{array}{l} \leftarrow \text{Multiplication of numerators} \\ \leftarrow \text{Multiplication of denominators} \end{array}$$

In the same manner, we multiply the rational numbers.
For example,

$$\frac{-2}{3} \times \frac{4}{5} = \frac{(-2) \times 4}{3 \times 5} = \frac{-8}{15} \quad \begin{array}{l} \leftarrow \text{Multiplication of numerators} \\ \leftarrow \text{Multiplication of denominators} \end{array}$$

Remember

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then their product is given by,

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

$$\frac{\text{Product of numerators}}{\text{Product of denominators}}$$

Example 11: Multiply $\frac{4}{5}$ by $\frac{-10}{3}$.

Solution: $\frac{4}{5} \times \frac{-10}{3} = \frac{4 \times (-10)}{5 \times 3}$ (Divide (–10) and 5 by 5 to get rational number in standard form)

$$= \frac{4 \times (-2)}{3} = \frac{-8}{3}$$

Example 12: Find the product of $\frac{-6}{11}$ and $\frac{-7}{9}$.

Solution:

$$\frac{-6}{11} \times \frac{-7}{9} = \frac{(-6) \times (-7)}{11 \times 9} \quad \leftarrow \text{Not in the standard form}$$

$$= \frac{(-2) \times (-7)}{11 \times 3} \quad \leftarrow \text{Dividing } -6 \text{ and } 9 \text{ by their common factor } 3$$

$$= \frac{14}{33}$$

> ***Note:***
>
> It is clear from above examples that the product of two rational numbers is a rational number.

PROPERTIES OF MULTIPLICATION OF RATIONAL NUMBERS

Multiplication of rational numbers has properties similar to those of multiplication of fractions.

1. **Verify that the product of two rational numbers $\frac{3}{5}$ and $\frac{(-4)}{7}$ remains the same even if the order is changed.**

$\frac{3}{5} \times \frac{(-4)}{7}$	$\frac{(-4)}{7} \times \frac{3}{5}$
$= \frac{3 \times (-4)}{5 \times 7}$	$= \frac{(-4) \times 3}{7 \times 5}$
$= \frac{-12}{35}$	$= \frac{-12}{35}$

Product is same

Property 1: **Product of two rational numbers remains the same even if we change their order, i.e. if x and y are rational numbers, then**

$$x \times y = y \times x.$$

This is commutative law of multiplication.

2. **Verify that the product of rational numbers $\frac{-3}{4}, \frac{5}{7}, \frac{-2}{11}$ remains the same even after changing the groupings.**

$\left(\frac{-3}{4} \times \frac{5}{7}\right) \times \left(\frac{-2}{11}\right)$	$\frac{-3}{4} \times \left(\frac{5}{7} \times \frac{-2}{11}\right)$

$$= \left(\frac{-3 \times 5}{4 \times 7}\right) \times \left(\frac{-2}{11}\right) \qquad = \frac{-3}{4} \times \left(\frac{5 \times (-2)}{7 \times 11}\right)$$

$$= \frac{-15}{28} \times \frac{(-2)}{11} \qquad = \frac{-3}{4} \times \frac{-10}{77}$$

$$= \frac{(-15) \times (-1)}{14 \times 11} \text{ (in lowest terms)} \qquad = \frac{(-3) \times (-10)}{4 \times 77} \text{ (in lowest terms)}$$

$$= \frac{15}{154} \qquad = \frac{(-3) \times (-5)}{2 \times 77}$$

$$= \frac{15}{154}$$

Product is same

Property 2: **Product remains the same even when we change the grouping of the rational numbers, i.e. if x, y and z are rational numbers, then**

$$(x \times y) \times z = x \times (y \times z)$$

This is associative law of multiplication.

3. Find the product of $\frac{-4}{7}$ and 0.

Now, $\frac{-4}{7} \times 0 = \frac{-4 \times 0}{7 \times 1} = \frac{-0}{7} = 0$

Similarly, $0 \times \frac{-4}{7} = \frac{0 \times (-4)}{1 \times 7} = \frac{0}{7} = 0$

Property 3: **Product of a rational number and zero is zero, i.e. if x is any rational number, then**

$$x \times 0 = 0 = 0 \times x$$

4. Find the product of $\frac{-15}{37}$ and 1.

Now, $\frac{-15}{37} \times 1 = \frac{-15}{37} \times \frac{1}{1} = \frac{-15 \times 1}{37 \times 1} = \frac{-15}{37}$

Similarly, $1 \times \frac{-15}{37} = \frac{1}{1} \times \frac{-15}{37} = \frac{1 \times -15}{37} = \frac{-15}{37}$

Property 4: **One multiplied by any rational number is the rational number itself, i.e. if x is a rational number, then**

$$x \times 1 = 1 \times x = x$$

i.e. 1 is the identity element under multiplication.

5. **Take three rational numbers, $\frac{2}{5}, \frac{5}{7}$ and $\frac{6}{15}$ and find out the value of $\frac{2}{5} \times \left(\frac{5}{7} + \frac{6}{15}\right)$ and $\left(\frac{2}{5} \times \frac{5}{7}\right) + \left(\frac{2}{5} \times \frac{6}{15}\right)$.**

$$\frac{2}{5} \times \left(\frac{5}{7} + \frac{6}{15}\right)$$
$$= \frac{2}{5} \times \left(\frac{75+42}{105}\right)$$
$$= \frac{2}{5} \times \left(\frac{117}{105}\right)$$
$$= \frac{2}{5} \times \frac{39}{35} \quad \text{(in lowest terms)}$$
$$= \frac{78}{175} \quad \text{(in lowest terms)}$$

$$\left(\frac{2}{5} \times \frac{5}{7}\right) + \left(\frac{2}{5} \times \frac{6}{15}\right)$$
$$= \frac{2 \times 5}{5 \times 7} + \frac{2 \times 6}{5 \times 15}$$
$$= \frac{2}{7} + \frac{2 \times 2}{5 \times 5} \quad \text{(in lowest terms)}$$
$$= \frac{2}{7} + \frac{4}{25}$$
$$= \frac{50+28}{175} = \frac{78}{175}$$

In both cases, the result is the same.

So, we can say $\frac{2}{5} \times \left(\frac{5}{7} + \frac{6}{15}\right) = \left(\frac{2}{5} \times \frac{5}{7}\right) + \left(\frac{2}{5} \times \frac{6}{15}\right)$

Property 5: **If x, y and z are rational numbers, then**

(i) $x \times (y + z) = x \times y + x \times z$ (ii) $x \times (y - z) = x \times y - x \times z$

This is distributive law of multiplication over addition.

We illustrate (ii) with the help of the following example.

Example 13: For rational numbers $x = \frac{5}{6}$, $y = \frac{-7}{3}$, $z = \frac{2}{9}$, verify that $x \times (y - z) = x \times y - x \times z$.

Solution:

$$x \times (y - z)$$
$$= \frac{5}{6} \times \left[\frac{-7}{3} - \frac{2}{9}\right]$$
$$= \frac{5}{6} \times \left[\frac{-7 \times 3 - 2}{9}\right]$$
$$= \frac{5}{6} \times \left[\frac{-21-2}{9}\right]$$
$$= \frac{5}{6} \times \left[\frac{-23}{9}\right]$$

$$x \times y - x \times z$$
$$= \frac{5}{6} \times \frac{-7}{3} - \frac{5}{6} \times \frac{2}{9}$$
$$= \frac{5 \times -7}{6 \times 3} - \frac{5 \times 2}{6 \times 9}$$
$$= \frac{-35}{18} - \frac{10}{54}$$
$$= \frac{-35 \times 3 - 10}{54}$$

$$= \frac{5 \times (-23)}{6 \times 9} \qquad = \frac{-105-10}{54}$$

$$= \frac{-115}{54} \qquad = \frac{-115}{54}$$

Therefore, $x \times (y - z) = x \times y - x \times z$

Worksheet 3

1. Multiply and express the result as a rational number in the standard form.

(i) $\frac{11}{7}$ by $\frac{-3}{8}$ (ii) $\frac{-7}{4}$ by $\frac{2}{3}$ (iii) $\frac{-3}{12}$ by -48

(iv) $\frac{-14}{9}$ by $\frac{-3}{7}$ (v) $\frac{23}{5}$ by $\frac{-25}{11}$ (vi) 7 by $\frac{-15}{63}$

2. For the following values of x and y, verify x × y = y × x.

(i) $x = \frac{7}{9}, y = \frac{3}{2}$ (ii) $x = \frac{-2}{7}, y = \frac{5}{8}$

(iii) $x = \frac{4}{9}, y = \frac{-5}{11}$ (iv) $x = \frac{-17}{48}, y = \frac{-96}{51}$

3. For the following values of x, y and z, find the products (x × y) × z and x × (y × z) and observe the result (x × y) × z = x × (y × z).

(i) $x = \frac{3}{5}, y = \frac{-7}{3}, z = \frac{8}{11}$ (ii) $x = \frac{-7}{11}, y = \frac{4}{5}, z = \frac{3}{8}$

(iii) $x = \frac{-4}{7}, y = \frac{-3}{8}, z = \frac{16}{5}$ (iv) $x = -3, y = \frac{-4}{9}, z = \frac{-7}{3}$

4. Verify the property x × (y + z) = x × y + x × z by taking—

(i) $x = \frac{1}{3}, y = \frac{1}{5}, z = \frac{1}{7}$ (ii) $x = \frac{-3}{7}, y = \frac{2}{5}, z = \frac{-4}{9}$

5. Show that–

$$\frac{-4}{3} \times \left(\frac{2}{5} + \frac{-7}{10}\right) = \left(\frac{-4}{3} \times \frac{2}{5}\right) + \left(\frac{-4}{3} \times \frac{-7}{10}\right)$$

6. Show that–

$$\frac{3}{5} \times \left(\frac{-1}{7} - \frac{5}{14}\right) = \left(\frac{3}{5} \times \frac{-1}{7}\right) - \left(\frac{3}{5} \times \frac{5}{14}\right)$$

7. Simplify and express the result in standard form.

(i) $-4 \times \left(\frac{7}{3} - \frac{9}{10}\right)$

(ii) $\frac{7}{3} \times \left(\frac{9}{8} + 3\right)$

(iii) $\left(\frac{-4}{3} + \frac{5}{7}\right) \times \frac{7}{9}$

(iv) $\left(\frac{5}{4} - \frac{6}{20}\right) \times \frac{8}{11}$

8. Fill in the blanks.

(i) $\frac{-4}{7} \times$ ______ $= \frac{-4}{7}$

(ii) $\frac{3}{8} \times$ ______ $= \frac{-3}{8}$

(iii) $\left(\frac{-1}{3} \times \frac{4}{5}\right) \times \frac{6}{7} =$ ______ $\times \left(\frac{4}{5} \times \frac{6}{7}\right)$

(iv) $\frac{5}{3} \times \left(\frac{-7}{8} \times \frac{11}{3}\right) = \left(\frac{5}{3} \times\right.$ ______ $\left.\right) \times \frac{11}{3}$

(v) $\frac{3}{7} \times \frac{-6}{11} = \frac{-6}{11} \times$ ______

(vi) $\frac{4}{3} \times$ ______ $= 0$

(vii) For any rational number x, $x \times 5 = x + x + \ldots\ldots\ldots$ ______ times.

(viii) $\frac{2}{3} \times \left(\frac{7}{5} - \frac{2}{9}\right) = \frac{2}{3} \times \frac{7}{5} -$ ______

(ix) $\frac{-5}{7} \times \frac{1}{3} + \frac{-5}{7} \times \frac{1}{6} = \frac{-5}{7} \times \left(\frac{1}{3} +\right.$ ______ $\left.\right)$

(x) $\frac{4}{7} \times \frac{2}{3} - \frac{4}{7} \times \frac{5}{6} = \frac{4}{7} \times ($ ______ $-$ ______ $)$

RECIPROCAL OF A RATIONAL NUMBER

Let us consider a rational number $\frac{9}{4}$. We try to find a rational number which when multiplied by $\frac{9}{4}$ gives us the result 1. In other words, let us try to fill in the blanks so that the statement $\frac{9}{4} \times$ ______ $= \frac{1}{1} = 1$.

You must fill in the blank by $\frac{4}{9}$ so that

$$\frac{9}{4} \times \frac{4}{9} = \frac{36}{36} = \frac{1}{1} = 1.$$

The product of $\frac{9}{4}$ and $\frac{4}{9}$ is 1.

$\frac{4}{9}$ is called the **reciprocal (multiplicative inverse)** of $\frac{9}{4}$.

Verify whether (i) $\frac{-4}{3}$ is reciprocal of $\frac{-3}{4}$ (ii) $\frac{-7}{3}$ is reciprocal of $\frac{3}{7}$.

(i) Find the product.

$$\frac{-4}{3} \times \frac{-3}{4} = \frac{(-4) \times (-3)}{3 \times 4} = \frac{12}{12} = 1$$

Hence, $\frac{-3}{4}$ is the reciprocal of $\frac{-4}{3}$.

(ii) Observe the product.

$$\frac{3}{7} \times \frac{-7}{3} = \frac{3 \times (-7)}{7 \times 3} = \frac{-21}{21} = -1$$

Since we are getting – 1, therefore, $\frac{-7}{3}$ is not the reciprocal of $\frac{3}{7}$.

Two non-zero numbers $\frac{a}{b}$ and $\frac{b}{a}$ are reciprocals of each other.

Note:

(i) To get the reciprocal of a given rational number, simply interchange the integers in the numerator and the denominator.

(ii) Zero has no reciprocal.

(iii) Reciprocal of 1 is 1.

(iv) If x is any non-zero rational number, then its reciprocal is denoted by x^{-1} which is equal to $\frac{1}{x}$.

Example 14: Find the reciprocals of the following rational numbers.

(i) $\frac{-3}{7}$ (ii) $\frac{3}{-7}$ (iii) $\frac{-3}{-7}$

Solution: By simply interchanging the numerator and denominator, we can find reciprocal.

(i) Reciprocal of $\frac{-3}{7}$ is $\frac{7}{-3}$.

(ii) Reciprocal of $\frac{3}{-7}$ is $\frac{-7}{3}$.

(iii) Reciprocal of $\frac{-3}{-7}$ is $\frac{-7}{-3}$.

Example 15: Verify $(x \times y)^{-1} = x^{-1} \times y^{-1}$, for $x = \frac{-2}{5}$, $y = \frac{3}{5}$.

Solution: $(x \times y)^{-1} = \left(\frac{-2}{5} \times \frac{3}{5}\right)^{-1} = \left(\frac{-2 \times 3}{5 \times 5}\right)^{-1} = \left(\frac{-6}{25}\right)^{-1} = \frac{25}{-6}$

Now, $x^{-1} \times y^{-1} = \frac{5}{-2} \times \frac{5}{3} = \frac{5 \times 5}{-2 \times 3} = \frac{26}{-6}$.

In both the cases the value is the same.

Hence, $(x \times y)^{-1} = x^{-1} \times y^{-1}$.

Example 16: Check the validity of the result, $(x + y)^{-1} \neq x^{-1} + y^{-1}$ for $x = \frac{1}{3}$, $y = \frac{-2}{7}$.

Solution:

$$(x + y)^{-1} = \left(\frac{1}{3} + \frac{-2}{7}\right)^{-1}$$

$$= \left[\frac{7 + (-6)}{21}\right]^{-1} = \left[\frac{7 - 6}{21}\right]^{-1} = \left[\frac{1}{21}\right]^{-1} = 21$$

Now, $x^{-1} + y^{-1} = \left(\frac{1}{3}\right)^{-1} + \left(\frac{-2}{7}\right)^{-1} = 3 + \frac{7}{-2}$

$$= \frac{3}{1} + \frac{-7}{2}$$

$$= \frac{6 - 7}{2} = \frac{-1}{2}$$

Hence, $(x + y)^{-1} \neq x^{-1} + y^{-1}$.

Worksheet 4

1. Find the reciprocals of:

(i) $\frac{-1}{5}$ (ii) 4 (iii) $\frac{11}{-12}$ (iv) $\frac{-2}{-19}$

2. Check, if the reciprocal of $\frac{-2}{3}$ is $\frac{3}{2}$?

3. **Check if the reciprocal of $\frac{-1}{5}$ is -5?**

4. **Verify that $(x - y)^{-1} \neq x^{-1} - y^{-1}$ by taking $x = \frac{-2}{7}$, $y = \frac{4}{7}$.**

   [Hint: Proceed as in Example 16.]

5. **Verify that $(x + y)^{-1} \neq x^{-1} + y^{-1}$ by taking $x = \frac{5}{9}$ and $y = \frac{-4}{3}$.**

6. **Verify that $(x \times y)^{-1} = x^{-1} \times y^{-1}$ by taking $x = \frac{-2}{3}$ and $y = \frac{-3}{4}$.**

7. **Fill in the blanks.**

   (i) The number ________ has no reciprocal.

   (ii) ________ and ________ are their own reciprocals.

   (iii) If a is the reciprocal of b, then b is the reciprocal of ________.

   (iv) $(11 \times 5)^{-1} = (11)^{-1} \times$ ________.

   (v) $\frac{-1}{8} \times$ ________ $= 1$

   (vi) ________ $\times \left(-5\frac{1}{3}\right) = 1$

DIVISION OF RATIONAL NUMBERS

Remember

Dividing one rational number by another except by zero, is the same as the multiplication of the first by the reciprocal of the second,

i.e. $x \div y = x \times y^{-1}$

We illustrate this with the help of examples.

Example 17: Divide $\frac{7}{3}$ by $\frac{5}{3}$.

Solution:

$$\frac{7}{3} \div \frac{5}{3} = \frac{7}{3} \times \left(\frac{5}{3}\right)^{-1}$$

$$= \frac{7}{3} \times \frac{3}{5} = \frac{7}{5}.$$

Example 18: Divide $\frac{2}{9}$ by -4.

Solution:

$$\frac{2}{9} \div (-4) = \frac{2}{9} \times (-4)^{-1}$$

$$= \frac{2}{9} \times \frac{1}{-4}$$

$$= \frac{-1}{18} \qquad \text{(Standard form)}$$

Example 19: The product of two rational numbers is $\frac{-25}{16}$. If one of the numbers is $\frac{-5}{4}$, find the other.

Solution: Product of two numbers $= \frac{-25}{16}$

One number $= \frac{-5}{4}$

We can write it as $\frac{-5}{4} \times \boxed{\text{other number}} = \frac{-25}{16}$

or other number $= \frac{-25}{16} \div \frac{-5}{4} = \frac{-25}{16} \times \left(\frac{-5}{4}\right)^{-1}$

$$= \frac{-25}{16} \times \frac{4}{-5}$$

$$= \frac{5 \times 1}{4 \times 1} = \frac{5}{4}$$

PROPERTIES OF DIVISION OF RATIONAL NUMBERS

1. Divide two rational numbers and find the result.

(i) $\frac{5}{3} \div \frac{-4}{3} = \frac{5}{3} \times \frac{3}{-4} = \frac{-5}{4}$

(ii) $\frac{4}{9} \div \frac{9}{4} = \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$

Similarly, (iii) $0 \div \frac{-4}{5} = 0$

Rational Numbers

Property 1: **Division of a rational number by another rational number except zero, is a rational number.**

2. **Find the quotient, if rational number $\frac{-3}{4}$ is divided by the same rational number.**

$$\frac{-3}{4} \div \frac{-3}{4} = \frac{-3}{4} \times \frac{4}{-3}$$
$$= 1$$

Property 2: **When a rational number (non-zero) is divided by the same rational number, the quotient is one.**

3. **Find the quotient when a rational number $\frac{-4}{5}$ is divided by 1.**

$$\frac{-4}{5} \div 1 = \frac{-4}{5} \times \frac{1}{1}$$
$$= \frac{-4}{5}$$

Property 3: **When a rational number is divided by 1, the quotient is the same rational number.**

4. **If $x = \frac{3}{2}$, $y = \frac{-4}{5}$, prove that $x \div y \neq y \div x$.**

$x \div y$	$y \div x$
$= \frac{3}{2} \div \frac{-4}{5}$	$= \frac{-4}{5} \div \frac{3}{2}$
$= \frac{3}{2} \times \frac{5}{-4}$	$= \frac{-4}{5} \times \frac{2}{3}$
$= \frac{15}{-8}$	$= \frac{-8}{15}$
$= \frac{-15}{8}$	

Not the same

Therefore, $x \div y \neq y \div x$.

Property 4: **If x and y are non-zero rational numbers, then in general, $x \div y \neq y \div x$ i.e. commutative property does not hold true for division.**

Note:

In fact $x \div y = \frac{1}{y \div x}$

5. **If $x = \frac{2}{3}$, $y = \frac{-4}{9}$, $z = \frac{5}{6}$, prove that $(x \div y) \div z \neq x \div (y \div z)$.**

$$(x \div y) \div z$$

$$= \left(\frac{2}{3} \div \frac{-4}{9}\right) \div \frac{5}{6}$$

$$= \left(\frac{2}{3} \times \frac{9}{-4}\right) \div \frac{5}{6}$$

$$= \frac{-3}{2} \div \frac{5}{6}$$

$$= \frac{-3}{2} \times \frac{6}{5}$$

$$= \frac{-3 \times 3}{5}$$

$$= \frac{-9}{5}$$

$$x \div (y \div z)$$

$$= \frac{2}{3} \div \left(\frac{-4}{9} \div \frac{5}{6}\right)$$

$$= \frac{2}{3} \div \left(\frac{-4}{9} \times \frac{6}{5}\right)$$

$$= \frac{2}{3} \div \left(\frac{-4 \times 2}{3 \times 5}\right)$$

$$= \frac{2}{3} \div \left(\frac{-8}{15}\right)$$

$$= \frac{2}{3} \times \frac{15}{-8}$$

$$= \frac{5}{-4} = \frac{-5}{4}$$

Not the same

Hence, $(x \div y) \div z \neq x \div (y \div z)$

Property 5: **If x, y and z are non-zero rational numbers, then, in general**

$$(x \div y) \div z \neq x \div (y \div z),$$

i.e. associative property does not hold true for division.

6. **Take $x = \frac{-2}{3}$, $y = \frac{5}{9}$, $z = \frac{-1}{6}$ and prove that $(x + y) \div z = x \div z + y \div z$ and $(x - y) \div z = x \div z - y \div z$.**

(i) First case

$$(x + y) \div z$$

$$= \left(\frac{-2}{3} + \frac{5}{9}\right) \div \frac{(-1)}{6}$$

$$= \left(\frac{-6 + 5}{9}\right) \div \frac{(-1)}{6}$$

$$= \frac{-1}{9} \div \frac{(-1)}{6}$$

$$= \frac{-1}{9} \times \frac{6}{-1} = \frac{2}{3}$$

$$(x \div z) + (y \div z)$$

$$= \frac{-2}{3} \div \frac{(-1)}{6} + \frac{5}{9} \div \frac{(-1)}{6}$$

$$= \frac{-2}{3} \times \frac{6}{(-1)} + \frac{5}{9} \times \frac{6}{(-1)}$$

$$= 4 - \frac{5 \times 2}{3} = \frac{4}{1} - \frac{10}{3}$$

$$= \frac{12 - 10}{3} = \frac{2}{3}$$

Thus, we have verified that $(x + y) \div z = x \div z + y \div z$.

(ii) Second case

$(x - y) \div z$	$x \div z - y \div z$
$= \left(\frac{-2}{3} - \frac{5}{9}\right) \div \frac{(-1)}{6}$	$= \frac{-2}{3} \div \frac{(-1)}{6} - \frac{5}{9} \div \frac{(-1)}{6}$
$= \left(\frac{-6-5}{9}\right) \div \frac{(-1)}{6}$	$= \frac{-2}{3} \times \frac{6}{(-1)} - \frac{5}{9} \times \frac{6}{(-1)}$
$= \frac{-11}{9} \times \frac{6}{(-1)}$	$= 2 \times 2 + \frac{5 \times 2}{3} = \frac{4}{1} + \frac{10}{3}$
$= \frac{11 \times 2}{3} = \frac{22}{3}$	$= \frac{12+10}{3} = \frac{22}{3}$

Here again, we have verified that $(x - y) \div z = x \div z - y \div z$.

Property 6: **If x, y and z are rational numbers, then $(x + y) \div z = x \div z + y \div z$ and $(x - y) \div z = x \div z - y \div z$**

7. If $x = \frac{2}{5}$, $y = \frac{-3}{10}$, $z = \frac{4}{15}$, prove $x \div (y + z) \neq x \div y + x \div z$.

$x \div (y + z)$	$x \div y + x \div z$
$= \frac{2}{5} \div \left(\frac{-3}{10} + \frac{4}{15}\right)$	$= \frac{2}{5} \div \frac{-3}{10} + \frac{2}{5} \div \frac{4}{15}$
$= \frac{2}{5} \div \left(\frac{-9+8}{30}\right)$	$= \frac{2}{5} \times \frac{10}{-3} + \frac{2}{5} \times \frac{15}{4}$
$= \frac{2}{5} \div \frac{-1}{30}$	$= \frac{-2 \times 2}{3} + \frac{1 \times 3}{1 \times 2}$
$= \frac{2}{5} \times \frac{30}{-1}$	$= \frac{-4}{3} + \frac{3}{2}$
$= -2 \times 6$	$= \frac{-8+9}{6}$
$= -12$	$= \frac{1}{6}$

Not the same

Hence, $x \div (y + z) \neq x \div y + x \div z$

Similarly, we may verify that $x \div (y - z) \neq x \div y - x \div z$.

Property 7: **For three non-zero rational numbers x, y and z, $x \div (y + z) \neq x \div y + x \div z$, i.e. distributive property does not hold true for division.**

Worksheet 5

1. **Divide.**

   (i) $\frac{2}{5}$ by $\frac{-1}{3}$ (ii) $\frac{-7}{4}$ by $\frac{1}{8}$ (iii) -10 by $\frac{1}{5}$ (iv) $\frac{1}{13}$ by -2

2. **By taking $x = \frac{3}{4}$ and $y = \frac{-5}{6}$, verify that $x \div y \neq y \div x$.**

3. **The product of two rational numbers is $\frac{-3}{7}$. If one of the number is $\frac{5}{21}$, find the other.**

4. **With what number should we multiply $\frac{-36}{35}$, so that the product be $\frac{-6}{5}$?**

5. **By taking $x = \frac{-5}{3}$, $y = \frac{2}{7}$ and $z = \frac{1}{-4}$, verify that–**

   (i) $x \div (y + z) \neq x \div y + x \div z$

   (ii) $x \div (x - z) \neq x \div y - x \div z$

   (iii) $(x + y) \div z = x \div z + y \div z$

6. **From a rope of the length 40 metres, a man cuts some equal sized pieces. How many pieces can be cut if each piece is of $\frac{4}{9}$ metres length?**

RATIONALS BETWEEN TWO RATIONAL NUMBERS

Let us find a rational number between $\frac{1}{4}$ and $\frac{3}{4}$.

For getting one rational number between $\frac{1}{4}$ and $\frac{3}{4}$, we add the two given rational numbers and then divide the sum by 2. That is,

$$\frac{1}{2}\left(\frac{1}{4}+\frac{3}{4}\right) = \frac{1}{2}\left(\frac{1+3}{4}\right)$$

$$= \frac{1}{2}\left(\frac{4}{4}\right) = \frac{1}{2}$$

which is a rational number lying between $\frac{1}{4}$ and $\frac{3}{4}$.

If x and y are two rational numbers, then $\frac{x+y}{2}$ is a rational number between x and y.

Example 20: Find three rational numbers between $\frac{1}{2}$ and $\frac{-1}{2}$.

Solution: **Step 1:** Find a rational number between $\frac{1}{2}$ and $\frac{-1}{2}$.

$$\frac{1}{2}\left[\frac{1}{2}+\left(\frac{-1}{2}\right)\right]=\frac{1}{2}\times 0=0$$

Step 2: Find a rational number between $\frac{1}{2}$ and 0.

$$\frac{1}{2}\left[\frac{1}{2}+0\right]=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

Step 3: Find a rational number between $\left(\frac{-1}{2}\right)$ and 0.

$$\frac{1}{2}\left[\frac{-1}{2}+0\right]=\frac{1}{2}\times\left(\frac{-1}{2}\right)=\frac{-1}{4}$$

Hence, 0, $\frac{1}{4}$, $\frac{-1}{4}$ are three rational numbers lying between $\frac{1}{2}$ and $\frac{-1}{2}$.

Note:

Between any two rational numbers, there are infinitely many rational numbers.

Worksheet 6

1. **Correct the following statements.**

   (i) Between two rational numbers, we can find only one rational number.

   (ii) Between two rational numbers, we can find as many integers as we like.

   (iii) Between two integers, we can find as many integers as we like.

2. **Find a rational number between:**

(i) 2 and 4

(ii) -2 and -6

(iii) $\frac{1}{4}$ and $\frac{-3}{4}$

(iv) $\frac{-2}{3}$ and $\frac{-7}{3}$

3. **Insert three rational numbers between:**

(i) $\frac{4}{13}$ and $\frac{1}{13}$

(ii) $\frac{-7}{10}$ and $\frac{11}{10}$

(iii) $\frac{-4}{3}$ and $\frac{-19}{3}$

(iv) $\frac{1}{8}$ and $\frac{-15}{8}$

4. **Find five rational numbers between:**

(i) $\frac{-4}{7}$ and $\left|\frac{-4}{7}\right|$

(ii) $\frac{-8}{3}$ and $\left|\frac{-8}{3}\right|$

VALUE BASED QUESTION

Rohit donated $\frac{1}{5}$ of his monthly income to an Non-Government Organisation (NGO) working for the education of the girl child, spent $\frac{1}{4}$ of his salary on food, $\frac{1}{3}$ on rent and $\frac{1}{15}$ on other expenses. He is left with ₹ 9000.

(a) Find Rohit's monthly salary.

(b) What values of Rohit are depicted here?

(c) Why is the education, specially for girls, important?

BRAIN TEASERS

1. A. **Tick (✓) the correct option.**

(a) The additive inverse of $\frac{-3}{4}$ is–

(i) $\frac{-3}{4}$ (ii) $\frac{-4}{3}$ (iii) $\frac{4}{3}$ (iv) $\frac{3}{4}$

(b) If x, y and z are rational numbers, then the property $(x + y) + z = x + (y + z)$ is known as–

(i) commutative property (ii) associative property

(iii) distributive property (iv) closure property

(c) $\frac{7}{12} \div \left(\frac{-7}{12}\right)$ is–

(i) 1 (ii) 7 (iii) -1 (iv) -7

(d) Identity element for subtraction of rational numbers is–

(i) 1 (ii) 0 (iii) -1 (iv) does not exist

(e) The multiplicative inverse of $6\frac{1}{3}$ is–

(i) $\frac{-19}{3}$ (ii) $\frac{-3}{19}$ (iii) $\frac{3}{19}$ (iv) $\frac{19}{3}$

B. Answer the following questions.

(a) Write all rational numbers whose absolute value is $\frac{5}{9}$.

(b) Find the reciprocal of $\frac{4}{5} \times \left(\frac{3}{-8}\right)$.

(c) What should be added to $\frac{-5}{11}$ to get $\frac{26}{33}$?

(d) Subtract $6\frac{2}{3}$ from the sum of $\frac{-3}{7}$ and 2.

(e) Find the value of $1 + \cfrac{1}{1 + \cfrac{1}{6}}$.

2. State whether the following statements are true or false. If false, justify your answer with an example.

(i) If $|x| = 0$, then x has no reciprocal. __________

(ii) If $x < y$ then $|x| < |y|$. __________

(iii) If $x < y$ then $x^{-1} < y^{-1}$ __________

(iv) The negative of a negative rational number is a positive rational number. __________

(v) Product of two rational numbers can never be an integer. ____________

(vi) Product of two integers is never a fraction. ____________

(vii) If x and y are two rational numbers such that x > y, then x – y is always a positive rational number. ____________

3. For $x = \frac{3}{4}$ and $y = \frac{-9}{8}$, insert a rational number between:

(i) $(x + y)^{-1}$ and $x^{-1} + y^{-1}$ (ii) $(x - y)^{-1}$ and $x^{-1} - y^{-1}$.

4. Verify that–

$(x \div y)^{-1} = x^{-1} \div y^{-1}$ by taking $x = \frac{-5}{11}$, $y = \frac{7}{3}$.

5. Verify that $|x + y| \le |x| + |y|$ by taking $x = \frac{2}{3}$, $y = \frac{-3}{5}$.

6. Find the reciprocals of:

(i) $\frac{2}{-5} \times \frac{3}{-7}$ (ii) $\frac{-4}{3} \times \frac{-5}{-8}$

7. Simplify.

(i) $\left|\frac{5}{7} - \frac{2}{3}\right| + \left|\frac{3}{14} - \frac{5}{7}\right|$ (ii) $\left(\frac{5}{11}\right)^{-1} - \frac{13}{5} + \frac{3}{15}$

(iii) $\frac{9}{5} \times \frac{-2}{27} + \frac{7}{30}$ (iv) $\frac{-7}{15} \div \left(\frac{50}{3}\right)^{-1}$

8. Divide.

(i) The sum of $\frac{5}{21}$ and $\frac{4}{7}$ by their difference.

(ii) The difference of $\frac{12}{5}$, $\frac{-16}{20}$ by their product.

9. Find reciprocal of $\frac{-2}{3} \times \frac{5}{7} + \frac{2}{9} \div \frac{1}{3} \times \frac{6}{7}$.

HOTS

1. A drum of kerosene oil is $\frac{3}{4}$ full. When 15 litres of oil is drawn from it, it is $\frac{7}{12}$ full. Find the total capacity of the drum.

2. Find the product of:

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{10}\right)$$

ENRICHMENT QUESTION

Complete the following magic square of multiplication.

$\frac{1}{81}\times\frac{1}{81}$		$\frac{1}{81}\times\frac{1}{9}$
	$\frac{1}{3}\times\frac{1}{81}$	
$\frac{1}{27}\times\frac{1}{3}$		$\frac{1}{3}\times\frac{1}{3}$

YOU MUST KNOW

1. If $\frac{a}{b}$ and $\frac{c}{d}$ are non-zero rational numbers, then–

   (i) $\frac{a}{b}+\frac{c}{d}=\frac{a\times d+b\times c}{b\times d}$

   (ii) $\frac{a}{b}-\frac{c}{d}=\frac{a\times d-b\times c}{b\times d}$

   (iii) $\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$

   (iv) $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{a\times d}{b\times c}$

2. If x, y and z are rational numbers, then–

   (i) $x + y$ is a rational number.

   (ii) $x + y = y + x$

   (iii) $0 + x = x + 0 = x$

   (iv) $x + (y + z) = (x + y) + z$

3. If x, y, and z are rational numbers, then–

   (i) $x - y$ is a rational number.

   (ii) $x - y \neq y - x$

   (iii) $x - 0 = x \neq 0 - x$

   (iv) $(x - y) - z \neq x - (y - z)$

4. If x, y, and z are rational numbers, then–

(i) $x \times y$ is a rational number.

(ii) $x \times y = y \times x$

(iii) $x \times 0 = 0 = 0 \times x$

(iv) $x \times 1 = x = 1 \times x$

(v) $(x \times y) \times z = x \times (y \times z)$

(vi) $x \times (y + z) = x \times y + x \times z$

(vii) $x \times (y - z) = x \times y - x \times z$

5. Two non-zero rational numbers $\frac{a}{b}$ and $\frac{b}{a}$ are reciprocals of each other.

6. If x, y and z are rational numbers, then–

(i) $x \div y$ is also a rational number, $y \neq 0$

(ii) $x \div 1 = x$

(iii) $x \div y \neq y \div x$ in general

(iv) $(x \div y) \div z \neq x \div (y \div z)$

(v) $x \div (y + z) \neq x \div y + x \div z$

(vi) $x \div (y - z) \neq x \div y - x \div z$

(vii) $(x + y) \div z = x \div z + y \div z$

(viii) $(x - y) \div z = x \div z - y \div z$

7. (i) $\frac{x + y}{2}$ is a rational number between two rational numbers x and y.

(ii) Between any two rational numbers, there are infinitely many rational numbers.

CHAPTER 3

RATIONAL NUMBERS AS DECIMALS

INTRODUCTION

Do you remember how to write fraction in decimal form?

$\frac{5}{10} = 0.5$ $\frac{13}{100} = 0.13$ $\frac{11}{10} = 1.1$

$\frac{7}{1000} = 0.007$ $\frac{231}{1000} = 0.231$

Any fraction having denominator as 10 or a power of 10 (i.e. 10, 100, 1000, . . .) can be easily represented in decimal form.

RATIONAL NUMBERS AS DECIMALS

Here, we shall study how to represent a rational number in decimal form, if the denominator is not 10 or a power of 10.

Let us do some examples.

Example 1: Convert $\frac{3}{4}$ in its decimal form.

Solution: Convert the denominator into 10 or power of 10 and write $\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$

Example 2: Convert the following in the decimal form.

(i) $\frac{4}{5}$ (ii) $-\frac{2}{25}$ (iii) $\frac{3}{125}$ (iv) $\frac{7}{20}$

Solution: (i) $\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10} = 0.8$

(ii) $\frac{-2}{25} = -\frac{2 \times 4}{25 \times 4} = -\frac{8}{100} = -0.08$

(iii) $\frac{3}{125} = \frac{3 \times 8}{125 \times 8} = \frac{24}{1000} = 0.024$

(iv) $\frac{7}{20} = \frac{7 \times 5}{20 \times 5} = \frac{35}{100} = 0.35$

Worksheet 1

1. Express the following rational numbers as decimals.

(i) $\frac{9}{8}$ (ii) $\frac{615}{125}$ (iii) $\frac{1}{16}$

(iv) $-\frac{3}{4}$ (v) $\frac{59}{200}$ (vi) $\frac{-24}{25}$

(vii) $\frac{-53}{250}$ (viii) $\frac{47}{400}$ (ix) $\frac{27}{800}$

(x) $\frac{139}{625}$ (xi) $\frac{3186}{1250}$ (xii) $\frac{133}{25}$

CONVERSION OF RATIONAL NUMBERS INTO DECIMALS BY LONG DIVISION METHOD

Every rational number can be represented in the form of decimal by using long division method. The representation can be either terminating or non-terminating but repeating.

I. Terminating Decimals

Consider the following examples.

Example 3: Express the following rational numbers as decimals by using long division method.

(i) $\frac{5}{8}$ (ii) $\frac{13}{5}$ (iii) $\frac{629}{125}$

Solution: To represent rational number in a decimal form, divide numerator by denominator.

(i)
$$\begin{array}{r} 0.625 \\ 8\overline{)5.0} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ \underline{0} \end{array}$$

(ii)
$$\begin{array}{r} 2.6 \\ 5\overline{)13} \\ \underline{10} \\ 30 \\ \underline{30} \\ \underline{0} \end{array}$$

(iii)
$$\begin{array}{r} 5.032 \\ 125\overline{)629} \\ \underline{625} \\ 400 \\ \underline{375} \\ 250 \\ \underline{250} \\ \underline{0} \end{array}$$

$\therefore \ \dfrac{5}{8} = 0.625$ $\qquad \therefore \ \dfrac{13}{5} = 2.6$ $\qquad \therefore \ \dfrac{629}{125} = 5.032$

Remember

Digits in the decimal part are called **decimal places.**

In the above examples when the numerator is divided by the denominator, we get the quotient with definite number of decimal places because the long division comes to an end after few steps.

Here, in each case, we get zero as the remainder and the quotient has a finite number of decimal places. Thus, the decimal obtained is called **Terminating Decimal**.

II. Non-Terminating Decimals

Let us consider the following examples.

Example 4: Convert the following rational numbers into decimal form.

(i) $\dfrac{1}{3}$ $\qquad$ (ii) $\dfrac{2}{7}$ $\qquad$ (iii) $\dfrac{25}{12}$

Solution: To represent $\dfrac{1}{3}$ as a decimal, we divide 1 by 3.

$$\begin{array}{r} 0.333... \\ 3\overline{)1.0} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

Therefore, decimal form of $\dfrac{1}{3} = 0.333\ldots$

Here, the remainder 1 keeps on repeating again and again and 3 in the quotient also keeps on repeating.

When a digit goes on repeating endlessly, we place a bar (–) over it. Here, 3 is getting repeated, so we can write it as $\overline{3}$. Thus,

$$\frac{1}{3} = 0.333\ldots = 0.\overline{3}$$

(ii) To represent $\frac{2}{7}$ as a decimal, we divide 2 by 7.

$$\begin{array}{r|l} & 0.285714\ldots \\ \hline 7 & 2.0 \\ & \underline{14} \\ & 60 \\ & \underline{56} \\ & 40 \\ & \underline{35} \\ & 50 \\ & \underline{49} \\ & 10 \\ & \underline{7} \\ & 30 \\ & \underline{28} \\ & 2 \end{array}$$

$$\therefore \frac{2}{7} = 0.285714\ldots = 0.\overline{285714}$$

Here, we have remainder as 2 which is just the same as the dividend. Therefore, after 4, the same digits, i.e. 2, 8, 5, 7, 1, 4 will keep on repeating again and again in the quotient.

(iii) To represent $\frac{25}{12}$ as a decimal, we divide 25 by 12.

$$\begin{array}{r|l} & 2.0833\ldots \\ \hline 12 & 25.0 \\ & \underline{24} \\ & 100 \\ & \underline{96} \\ & 40 \\ & \underline{36} \\ & 4 \end{array}$$

$\therefore \ \frac{25}{12} = 2.0833\ ...\ = 2.08\overline{3}$

Here, we have remainder as 4 which has appeared after the second step and then it is repeated. Therefore, the digit 3 in the quotient will keep on repeating.

From the above examples, we find that the division process does not terminate. Such numbers are called **Non-Terminating Repeating Decimals**.

So far we have converted positive rational numbers into decimal form. Now, we shall discuss how to convert negative rational numbers into decimal form.

Example 5: Find the decimal representation of the following rational numbers:

(i) $\frac{-5}{4}$ (ii) $\frac{-19}{7}$

Solution: (i) We know that $\frac{5}{4}$ in decimal form is represented as 1.25

Therefore, $\frac{-5}{4} = -1.25$

(ii) We also know that $\frac{19}{7}$ in decimal form is represented as $2.\overline{714285}$

Therefore, $\frac{-19}{7} = -2.\overline{714285}$

Now, let us see under which condition the decimal representation of a rational number terminates and under which condition it does not terminate. We take some examples to explain this.

Example 6: Find out whether the decimal representation of a rational number is terminating or non-terminating.

Solution:

$\frac{3}{2} = 1.5$ $\frac{3}{4} = 0.75$ $\frac{1}{8} = 0.125$

$\frac{2}{5} = 0.4$ $\frac{6}{25} = 0.24$ $\frac{2}{125} = 0.016$

$\frac{1}{10} = 0.1$ $\frac{1}{100} = 0.01$ $\frac{1}{1000} = 0.001$

To find out why we have terminating decimals in all the above examples, we observe prime factors of all the denominators.

$2 = 2$	$4 = 2 \times 2$	$8 = 2 \times 2 \times 2$
$5 = 5$	$25 = 5 \times 5$	$125 = 5 \times 5 \times 5$
$10 = 2 \times 5$	$100 = 2 \times 2 \times 5 \times 5$	$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

2 and 5 are the prime factors of all the denominators. Therefore, if we have only 2 and 5 as the prime factors of the denominator of a rational number in the lowest form, it will have terminating decimal representation. But, if the prime factors of the denominator are also other than 2 and 5, the decimal representation of that rational number (in the lowest form) will be a non-terminating repeating decimal.

Example 7: Without actual division, determine which of the following rational numbers have a terminating decimal representation?

(i) $\frac{21}{128}$ (ii) $\frac{27}{125}$ (iii) $\frac{39}{24}$ (iv) $-\frac{17}{90}$

Solution: We may note that all the above given numbers $\left(\text{except } \frac{39}{24}\right)$ are in the lowest form.

(i) The denominator of $\frac{21}{128}$ is 128.

$$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

The prime factor of 128 is 2 seven times.

Therefore, $\frac{21}{128}$ has a terminating decimal representation.

(ii) The denominator of $\frac{27}{125}$ is 125.

$$125 = 5 \times 5 \times 5$$

The prime factor of 125 is 5 three times.

Therefore, $\frac{27}{125}$ has a terminating decimal representation.

(iii) The denominator of $\frac{39}{24}$ is 24.

$$24 = 2 \times 2 \times 2 \times 3$$

The prime factors of 24 are 2 and 3. One of the factors is other than 2 and 5. But the rational number is not in its lowest form.

In fact $\frac{39}{24} = \frac{13 \times 3}{8 \times 3} = \frac{13}{8}$, whose denominator $8 = 2 \times 2 \times 2$.

Therefore, $\frac{39}{24}$ has a terminating decimal representation.

(iv) The denominator of $\frac{17}{90}$ is 90.

$$90 = 2 \times 3 \times 3 \times 5.$$

The prime factors of 90 are 2, 3 and 5.

One of the factors is other than 2 and 5.

Therefore, $\frac{17}{90}$ will not have a terminating decimal representation.

Worksheet 2

1. Express the following rational numbers as decimals by using long division method.

(i) $\frac{21}{16}$ (ii) $\frac{129}{25}$ (iii) $\frac{17}{200}$ (iv) $\frac{5}{11}$

(v) $\frac{22}{7}$ (vi) $\frac{31}{27}$ (vii) $\frac{2}{15}$ (viii) $\frac{63}{55}$

2. Without actual division, determine which of the following rational numbers have a terminating decimal representation and which have a non-terminating decimal representation.

(i) $\frac{11}{4}$ (ii) $\frac{13}{80}$ (iii) $\frac{15}{11}$ (iv) $\frac{22}{7}$

(v) $\frac{29}{250}$ (vi) $\frac{37}{21}$ (vii) $\frac{49}{14}$ (viii) $\frac{126}{45}$

3. Find the decimal representation of the following rational numbers.

(i) $\frac{-27}{4}$ (ii) $\frac{-37}{60}$ (iii) $\frac{-18}{125}$ (iv) $\frac{-15}{8}$

4. If the number $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$ is expressed as a decimal, will it be terminating or non-terminating? Justify your answer.

5. Justify the following statements as True or False.

(i) $\frac{22}{7}$ can be represented as a terminating decimal. __________

(ii) $\frac{51}{512}$ can be represented as a terminating decimal. __________

(iii) $\frac{19}{45}$ can be represented as a non-terminating repeating decimal. ______

(iv) $\frac{3}{17}$ cannot be represented as a non-terminating repeating decimal. ______

(v) If $\frac{3}{2}$ and $\frac{7}{5}$ are terminating decimals, then $\frac{3}{2} + \frac{7}{5}$ is also a terminating decimal. ______

(vi) If $\frac{1}{4}$ and $\frac{1}{5}$ both have terminating decimal representation, then $\frac{1}{4} \times \frac{1}{5}$ also has a terminating decimal representation. ______

CONVERSION OF TERMINATING DECIMALS INTO RATIONAL NUMBERS

In this section, we shall first discuss the process of converting a given decimal number into a rational number.

We take the following example to understand the process of conversion of a terminating decimal into the form $\frac{p}{q}$.

Example 7: Convert 1.2 in the form $\frac{p}{q}$.

Solution: Count the number of decimal places. In this case it is one.

Write the given number without the decimal point, i.e. 12.

Write a number with 12 in the numerator. The denominator is one followed by as many zeroes as is the number of decimal places in the given number.

In this case, the denominator will be 10.

Therefore, the required number is $\frac{12}{10}$.

Therefore, $1.2 = \frac{12}{10} = \frac{6}{5}$ (in the lowest form)

1 . 2

$$\frac{1\,2}{10} = \frac{12}{10}$$

Example 8: Express the following decimals in the form $\frac{p}{q}$.

(i) 0.4 (ii) 3.75 (iii) 1.025 (iv) 56.875 (v) – 2.56 (vi) – 0.25

Solution: (i) $0.4 = \frac{4}{10} = \frac{2}{5}$

(ii) $3.75 = \frac{375}{100} = \frac{15}{4}$

(iii) $1.025 = \frac{1025}{1000} = \frac{41}{40}$

(iv) $56.875 = \frac{56875}{1000} = \frac{455}{8}$

(v) $-2.56 = \frac{-256}{100} = \frac{-64}{25}$

(vi) $-0.25 = \frac{-25}{100} = \frac{-1}{4}$

Let us perform some operations on Decimal Numbers.

Example 9: Add 15.1, 12.03 and 7.209

Solution: First we convert these unlike decimal numbers into like decimal numbers then add as shown below.

$$\begin{array}{r} 15.100 \\ 12.030 \\ +\quad 7.209 \\ \hline 34.339 \\ \hline \end{array}$$

Example 10: Subtract 5.012 from 12.01

Solution: Convert into like decimal numbers and then subtract.

$$\begin{array}{r} 12.010 \\ -\quad 5.012 \\ \hline 6.998 \\ \hline \end{array}$$

Example 11: Multiply:

(i) 2.4 by 3.5 (ii) 4.8 by 1.84

Solution: (i) 2.4×3.5

First multiply 24 and 35 without decimal point.

$$\begin{array}{r} 24 \\ \times\quad 35 \\ \hline 120 \\ +\quad 720 \\ \hline 840 \\ \hline \end{array}$$

$24 \times 35 = 840$

$2.4 \times 3.5 = 8.40 = 8.4$

We find that the sum of decimal places in the given numbers is 1 + 1 = 2. So, the required product is 8.40 or 8.4.

(ii) 4.8 by 1.84

$$\begin{array}{r} 184 \\ \times \quad 48 \\ \hline 1472 \\ + \; 7360 \\ \hline 8832 \\ \hline \end{array}$$

$48 \times 184 = 8832$

$4.8 \times 1.84 = 8.832$

We find that the sum of the number of decimal places in the given two numbers is 1 + 2 = 3. So, the required product is 8.832.

Example 12: Divide:

(i) 32.768 by 8 (ii) 6.25 by 0.5.

Solution: (i) $32.768 \div 8$

$$\begin{array}{r} 4.096 \\ 8\overline{)32.768} \\ \underline{32} \\ 76 \\ \underline{72} \\ 48 \\ \underline{48} \\ 0 \\ \hline \end{array}$$

Therefore, $32.768 \div 8 = 4.096$

(ii) $6.25 \div 0.5$

Now, $\dfrac{6.25}{0.5} = \dfrac{625}{10\cancel{0}} \times \dfrac{1\cancel{0}}{5}$

$= \dfrac{625}{50} = \dfrac{125}{10} = 12.5$

Example 13: Simplify and express the result in the decimal form.

$\dfrac{1}{5} + \dfrac{3}{10} + \dfrac{4}{25}$

Solution: $\frac{1}{5}+\frac{3}{10}+\frac{4}{25}=\frac{10+15+8}{50}$

$$=\frac{33}{50}=0.66$$

$$\begin{array}{r} 0.66 \\ 50\overline{)33.0} \\ \underline{300} \\ 300 \\ \underline{300} \\ \underline{0} \end{array}$$

Worksheet 3

1. Express the following decimals as rational numbers in standard form.

(i) 0.25 (ii) – 0.052 (iii) 7.50

(iv) – 2.15 (v) 0.036 (vi) – 9.6

(vii) 31.25 (viii) 16.32 (ix) 0.107

2. Add the following decimals.

(i) 2.5, 7.51 and 11.501 (ii) 12, 5.96 and 3.076 (iii) 9.08, 19.76 and 20.54

(iv) 3.009, 0.592 and 14.745 (v) 19, 9.5, 12.06 and 17.921

3. Compute the following products of decimals.

(i) 2.9×3.5 (ii) 37×12.76 (iii) 0.84×8.8

(iv) 2.56×11.09 (v) $12.4 \times 15.7 \times 13.2$

4. Compute the following divisions.

(i) $59.049 \div 9$ (ii) $6.4 \div 0.2$ (iii) $0.015 \div 3$

(iv) $0.014 \div 12$ (v) $0.02472 \div 0.008$ (vi) $51.51 \div 0.17$

5. Evaluate the following:

(i) $25.75 + 2.09 - 13.6$ (ii) $37 - 16.58 + 12.25$

(iii) $42.7 - 11 - 9.025 + 2.16$ (iv) $(6.05 + 5.01) - (12.5 - 0.09)$

(v) $182.3 + 12.65 - 0.23 - 10.71$

6. Simplify and express the result as a rational number in its lowest terms.

(i) $\frac{1}{2}+\frac{1}{5}+6.25 \div 0.25$ (ii) $\frac{2}{5}-\frac{1}{4}+(8.1 \times 2.7) \div 0.09$

(iii) $1.44 \times (144 \div 12) - 0.225 + 3.276$ (iv) $\frac{1}{7} \times 0.049+\frac{3}{8}-\frac{7}{20}$

(v) $5 \times 0.16 - 0.52 + 8.263$ (vi) $\frac{2}{5} \times \frac{3}{4}+\frac{1}{25} \times \frac{1}{2}-\frac{2}{10} \times \frac{1}{5}$

VALUE BASED QUESTIONS

1. **Megha bought a book for ₹ $112\frac{1}{2}$ from a shop. She gave 500 rupee note to the shopkeeper and got the balance back. But, she realised that the shopkeeper had given her ₹ 72 extra. Megha returned the extra money and had a feeling of great satisfaction.**
   - (a) How much money had the shopkeeper returned to Megha?
   - (b) What values did Megha exhibit in the above situation?
2. **Raman had to cover a distance of 30 km to reach his grandmother's house. He covered 11.25 km by bus, 7.083 km by auto and rest by foot.**
   - (a) How much distance did Raman cover by foot?
   - (b) How is Raman benefitted if he walks down to any of his destination? In what ways does it effect out environment?

BRAIN TEASERS

1. **A. Tick (✓) the correct option.**

(a) 0.225 expressed as a rational number is–

(i) $\frac{1}{4}$ (ii) $\frac{45}{210}$ (iii) $\frac{9}{40}$ (iv) $\frac{225}{999}$

(b) A rational number $\frac{p}{q}$ can be expressed as a terminating decimal if q has no prime factor other than–

(i) 2, 3 (ii) 2, 5 (iii) 3, 5 (iv) 2, 3, 5

(c) $-7\frac{8}{100}$ expressed as a decimal number is–

(i) – 7.800 (ii) 7.008 (iii) – 7.008 (iv) – 7.08

(d) $4.01\overline{325}$ is equal to–

(i) 4.013252525... (ii) 4.0132555...

(iii) 4.0132501325... (iv) 4.0130132525...

(e) The quotient when 0.00639 is divided by 0.213 is–

(i) 3 (ii) 0.3 (iii) 0.03 (iv) 0.003

B. Answer the following questions.

(a) Without actual division, determine if $\frac{-28}{250}$ is terminating or non-terminating decimal number.

(b) Convert $\frac{-113}{7}$ to decimals.

(c) What should be subtracted from – 15.834 to get 3.476?

(d) Express 4.82 as rational number in standard form.

(e) Find the value of 16.016 ÷ 0.4.

2. Convert the following rational numbers into decimals.

(i) $\frac{259}{3}$ (ii) $\frac{19256}{11}$ (iii) $\frac{15735}{80}$

(iv) $\frac{27}{7}$ (v) $\frac{758}{1250}$ (vi) $\frac{15625}{12}$

3. Find the decimal representation of the following rational numbers.

(i) $-\frac{12}{13}$ (ii) $-\frac{1525}{50}$ (iii) $-\frac{127}{7}$ (iv) $-\frac{539}{80}$

4. Simplify the following expressions.

(i) 3.2 + 16.09 + 26.305 – 1.232 (ii) – 5.7 + 13.20 – 15.009 + 0.02

(iii) (0.357 + 0.96) – (3.25 – 2.79) (iv) 15 + 2.57 – 23.07 – 5.003

5. Without actual division, determine which of the following rational numbers have a terminating decimal representation.

(i) $\frac{327}{125}$ (ii) $\frac{99}{800}$ (iii) $\frac{17}{1250}$ (iv) $\frac{29}{200}$

(v) $\frac{135}{1625}$ (vi) $\frac{1276}{680}$ (vii) $\frac{22}{190}$ (viii) $\frac{11}{750}$

6. Simplify the following and express the result as decimals.

(i) 2.7 × 1.5 × 2.1 (ii) 12 × 13.6 × 0.25

(iii) 3.25 × 72.6 (iv) (156.25 ÷ 0.025) × 0.02 – 5.2

(v) (75.05 ÷ 0.05) × 0.001 + 2.351

7. **Simplify and express the result as a rational number in its lowest form.**

(i) $3.125 \div 0.125 + 0.50 - 0.225$ (ii) $\dfrac{0.4 \times 0.04 \times 0.005}{0.1 \times 10 \times 0.001} - \dfrac{1}{2} + \dfrac{1}{5}$

(iii) $\dfrac{0.144 \div 1.2}{0.016 \div 0.02} + \dfrac{7}{5} - \dfrac{21}{8}$.

HOTS

Perimeter of a rectangle is 2.4 m less than $\frac{2}{5}$ of the perimeter of a square.

If the perimeter of the square is 40 m, find the length and breadth of the rectangle given that breadth is $\frac{1}{3}$ of the length.

ENRICHMENT QUESTION

There is an interesting pattern in the following:

$$\frac{1}{7} = 0.\overline{142857}$$
$$\frac{2}{7} = 0.\overline{285714}$$
$$\frac{3}{7} = 0.\overline{428571}$$
$$\frac{4}{7} = 0.\overline{571428}$$
$$\frac{5}{7} = 0.\overline{714285}$$
$$\frac{6}{7} = 0.\overline{857142}$$

$$\frac{3}{7} + \frac{4}{7} = 0.\overline{9} \rightarrow \frac{2}{7} + \frac{5}{7} = 0.\overline{9} \rightarrow \frac{1}{7} + \frac{6}{7} = 0.\overline{9}$$

You will question why the left hand side in each case is 1, but, the right hand side is $0.\overline{9}$? (You will learn about this in higher classes).

Notice they all have only the digits 142857, each starting with a different digit but in the same order.

Try finding out the repeating part of the decimal for $\frac{1}{13}$. What do you notice?

YOU MUST KNOW

1. Every rational number can be represented as a decimal.
2. The decimal representation of a rational number is either terminating or non-terminating but repeating.
3. Decimal numbers having a finite number of decimal places are known as terminating decimal numbers.
4. Decimal numbers having an infinite number of decimal places are known as non-terminating decimal numbers.
5. Decimal numbers having an infinite number of decimal places and a set of digits in the decimal places that repeat are known as non-terminating repeating decimal numbers.
6. If the denominator of a rational number in the standard form has 2 or 5 or both as the only prime factors, then it can be represented as a terminating decimal.
7. If the denominator of a rational number in the standard form has prime factors other than 2 and 5, then it cannot be represented as a terminating decimal. In fact, it is a non-terminating but repeating decimal.

Class-VII

The LIVING WORLD

(A Book of Science & Technology)

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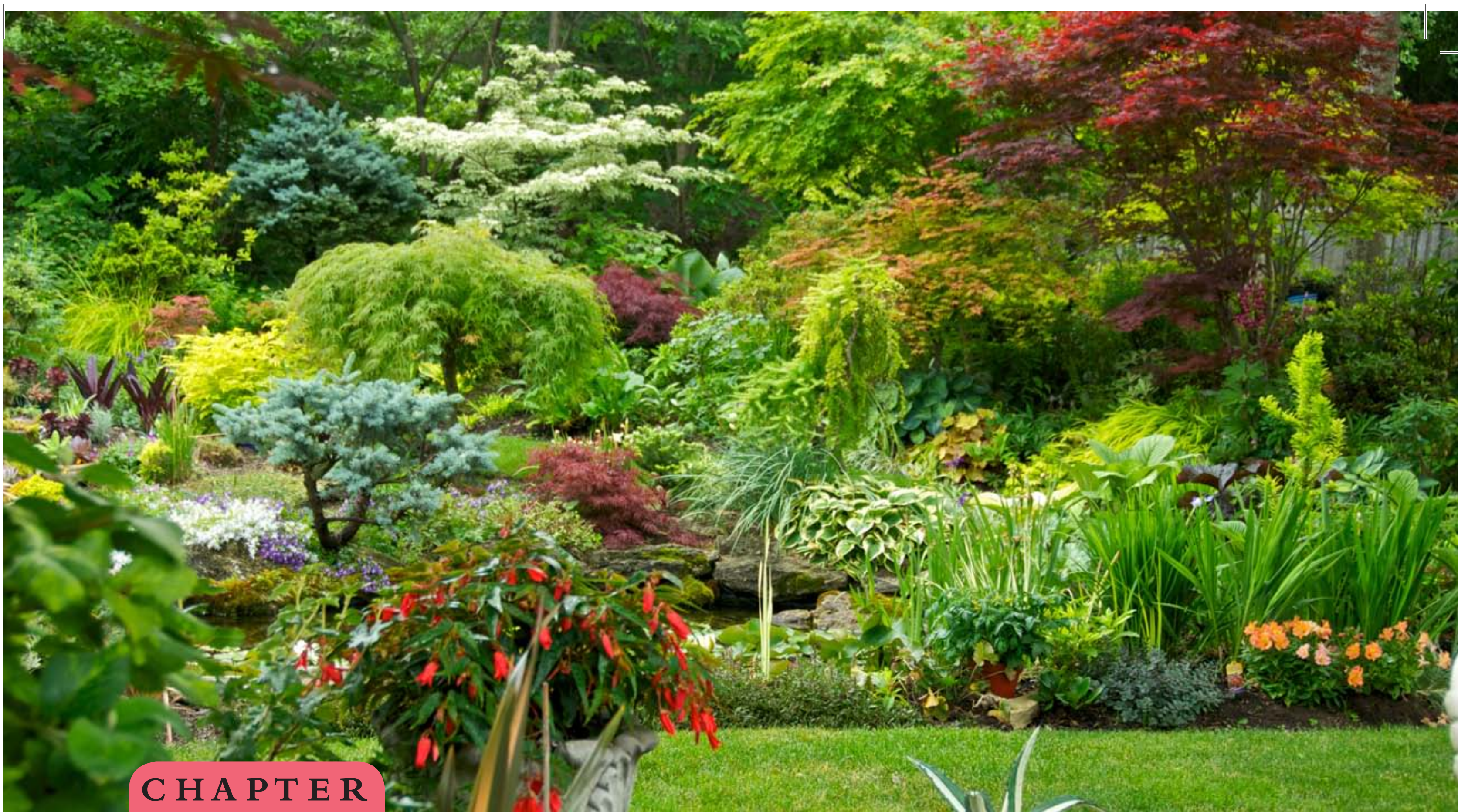
CONTENTS



CHAPTER

1 Nutrition in Living Organisms—Plants

In Class-VI, we have already learnt that food is essential for all living organisms. We also learnt that carbohydrates, proteins, fats, vitamins and minerals are all important components of our food. These components of food are necessary for our body and are called **nutrients**. The nutrients enable living organisms to build their bodies, to grow, to repair damaged parts of their bodies and to provide the energy to carry out life processes.

Nutrients are 'taken in' through their food by living organisms and are utilised in their bodies. This process of obtaining, and utilising food by an organism, is known as **nutrition**. The process of obtaining food is not the same in all organisms. On the basis of food habits, the modes of obtaining the required nutrition, by the body, have been divided into the following two categories:

- **Autotrophic Nutrition**

  It is the mode of nutrition in which organisms can make their own food from simple raw materials. All green plants and some bacteria are **autotrophs.** (In Greek, *auto* = self, *trophe* = nutrition).

- **Heterotrophic Nutrition**

  It is the mode of nutrition in which organisms cannot prepare their food on their own and depend on others for it. All animals, and a few plants, are **heterotrophs.** (In Greek, *heterone* = (an) other)

I can
prepare my own food by using
water and carbon dioxide in the presence of
sunlight that is captured by chlorophyll.

I cannot
prepare food on my own.
I depend on plants for food.

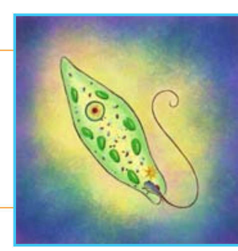
Let us study the following flow chart.

Modes of Nutrition

- **Autotrophic Nutrition** (All green plants)
- **Heterotrophic Nutrition** (Animals, Man, Non-green plants)
  - **Saprotrophic Nutrition** e.g. Fungi, Mushroom, Bread mould, and some Bacteria
  - **Parasitic Nutrition** e.g. Dodder
  - **Holozoic Nutrition** e.g. *Amoeba*, Humans

Do You Know?

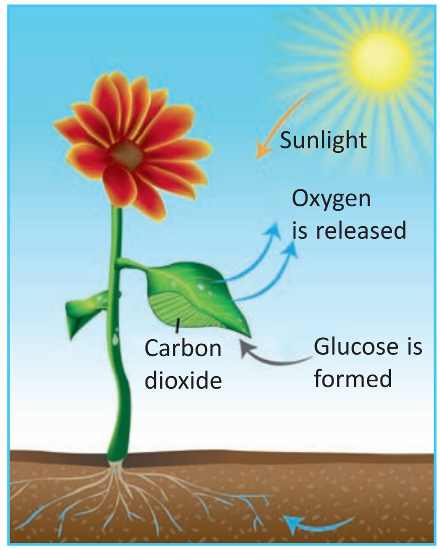
Euglena is an organism that shows both autotrophic and heterotrophic modes of nutrition. It has both plant and animal-like features.



Photosynthesis–Food Making Process in Plants

The synthesis of food in plants occurs in their leaves. Hence, leaves are called the **food factories** of the plants. The leaves have a green pigment called **chlorophyll**. It helps leaves to capture the energy of the sunlight. This energy is used by the plants to synthesise their food using carbon dioxide and water. This process is called **photosynthesis** (**photo** = light, **synthesis** = to combine) as it takes place in the presence of sunlight. This process can be written in the form of the following equation:

$$\text{carbon dioxide} + \text{water} \xrightarrow[\text{chlorophyll}]{\text{sunlight}} \underset{\text{(glucose)}}{\text{carbohydrate}} + \text{oxygen}$$



Photosynthesis

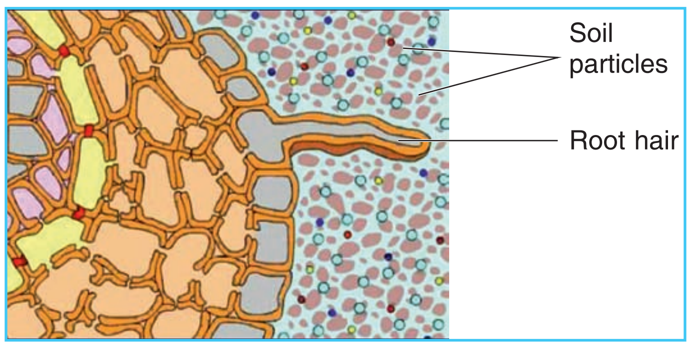
Raw Materials for Photosynthesis

From the above equation, it is clear that carbon dioxide and water are the raw materials for photosynthesis. For this process, chlorophyll and presence of sunlight/light are also necessary. Since food is synthesised in leaves, all the raw materials need to reach there.

Water and minerals (From soil)	← **Raw materials for photosynthesis** →	Carbon dioxide (From air)

Water and Minerals

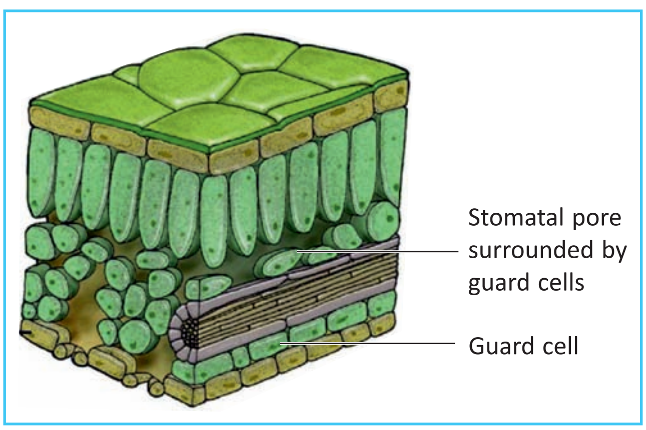
These are absorbed by the roots from the soil. From here, water and minerals are transported to other parts of the plant by the 'vessels'. **Vessels** are tubes that run throughout the root, the stem, the branches and the leaves. You will learn more about this in Chapter 8.



Roots absorb water and minerals from soil

Carbon dioxide

Plants take carbon dioxide from the atmosphere. Carbon dioxide enters the leaves through tiny pores present on the surface of leaves. Such pores are called **stomata**. The stomata are surrounded by special cells called **guard cells**.



Enlarged portion of leaf epidermis showing stomata

Activity 1

Take a potted plant. Apply a thin coat of vaseline on both sides of a leaf. Observe the plant for a few days. While all the other leaves remain green, the one, coated with vaseline, becomes yellow and falls off. This happens because the stomata of such a leaf get blocked. Such a leaf, cannot, therefore, take gases (like carbon dioxide and oxygen) from the atmosphere.

- ### Sunlight

  Sunlight is the light and energy that comes from the Sun. During photosynthesis the plants use the energy of sunlight to prepare food. That is why the food making process, in plants, is called **photosynthesis**. (Photo = light, synthesis = to combine)

- ### Chlorophyll

  The leaves are green due to the presence of a pigment—chlorophyll. It helps the leaves to capture solar energy. This energy is used to prepare food from carbon dioxide and water.

Do You Know?

Some plants have leaves that are not green in colour. Such leaves contain chlorophyll but the green colour is masked due to the presence of other coloured pigments. The presence of additional pigments causes other leaf colours, such as red in coleus and purple in red cabbage. However, such leaves can still perform photosynthesis.

However, some variegated leaves have yellow patches. Such yellow areas on the leaf do not contain any chlorophyll and hence, cannot perform photosynthesis.

Photosynthesis is a unique process. It is this process that supplies food, directly or indirectly, for all living organisms. The energy of the sun, thus, gets passed on to all organisms through plants. Plants also provide oxygen, needed by all living organisms, for respiration. Can you imagine life on earth in the absence of photosynthesis?

Do You Know?

Both deer and lion depend on plants. If there were no plants, deer would not survive and if there were no animals, like deer, the lions, too, would die. Plants, in turn, depend on solar energy. Hence, solar energy is the ultimate source of energy for all living organisms.

Products of Photosynthesis

The initial product of photosynthesis is a carbohydrate—glucose. It next gets converted to starch whose presence, in the leaves, indicates the occurrence of photosynthesis. Carbohydrates contain carbon, hydrogen and oxygen. Some carbohydrates are also converted to proteins and fats. Besides carbon, hydrogen and oxygen, proteins also contain nitrogen. Now where does this nitrogen come from? Nitrogen is present in the air but plants cannot use this nitrogen directly. Some bacteria, present in the soil, convert gaseous nitrogen into its usable form which is soluble and is, therefore, absorbed by roots along with water. Roots are also able to absorb nitrogenous compounds, present in fertilisers, that are added to the soil.

Other Modes of Nutrition in Plants

Some plants cannot synthesise their own food because they do not contain chlorophyll. Such plants depend on food produced by other plants. Their mode of nutrition is, therefore, **heterotrophic**. One such plant is *Cuscuta* (*amarbel*, dodder). It can be observed as a yellowish thread-like structure, without leaves, growing on other plants. *Cuscuta* is a **parasite** since it derives its nutrition from some other living organism and causes harm to that organism. The plant, on which it grows, is known as 'the **host**.'

Cuscuta-Dodder growing on a bush

Pitcher plant

Have you heard of insect-eating plants? There are plants that feed on insects for their nitrogen requirements. Some parts of such plants get modified to trap insects. For example, the leaf, of the pitcher plant, gets modified to form a pitcher with a lid. The lid is able to open and close the mouth of the pitcher. The pitcher is lined with downward-pointing hairs. When an insect enters, it cannot climb back out against the hairs and ultimately falls to the bottom of the leaf, and gets digested by the juices present there. Such insect-eating plants are called **insectivorous plants**.

Modes of Nutrition for Other Organisms

Saprotrophic Mode of Nutrition

'*Sapros*' means rotten and 'trophic' means food. **Saprotrophic nutrition** is the process in which the organisms feed on dead and decaying matter. The food gets digested outside the cells, or sometimes, even outside the body of the organism. This type of digestion is called **extracellular digestion**. The organism secretes digestive juices directly onto the food. These digestive juices make the food soluble; the organism then directly absorbs it. Some organisms, which have saprotrophic nutrition, are *Rhizopus* (bread mould), *Mucor* (pin mould), Yeast, *Agaricus* (mushroom) and many bacteria.

Do You Know?

You must have observed (i) a white cottony growth on leather articles in humid weather (ii) mushrooms growing on rotting wood and (iii) greenish-blue patches on rotting fruits. A cottony growth, developing into coloured patches, is a common occurrence on stale bread. These organisms belong to the group of fungi and bacteria, and they exhibit the saprotrohic mode of nutrition.

Symbiotic Relationship

Sometimes two organisms live in close association and develop a relationship that is beneficial to both. This is called **symbiotic relationship**. (In Greek, *symbion* = "to live together"). Some algae and fungi live in the roots of trees. They receive shelter and nutrition from the tree; in return, they help the trees to absorb water and minerals more efficiently.

Lichen is a living partnership between a fungus and an alga. The fungus absorbs water and provides shelter. The alga prepares food by photosynthesis.

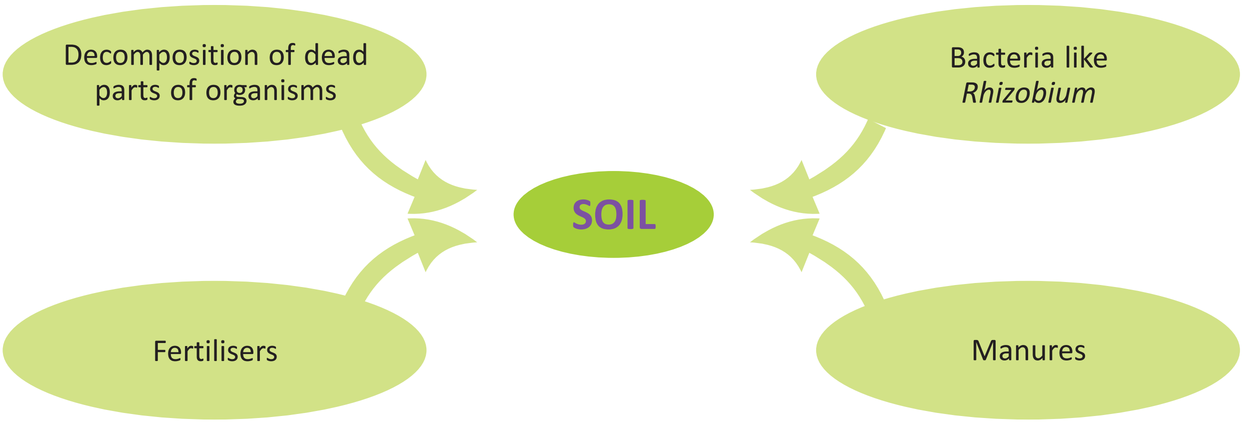
Lichens growing on rock

Rhizobium is a bacterium that lives in the roots of leguminous plants. It converts nitrogen, from the atmosphere, into a usable form that can be utilised by the plants. The plants, in turn, provide food and shelter to the bacterium.

Leguminous plant showing root nodules

How are Nutrients Replenished in the Soil?

Plants remove nutrients from the soil as they grow. These nutrients need to be reintroduced into the soil so that the soil remains productive. Farmers usually enrich the soil by adding manures and fertilisers; these are materials that contain one or more of the nutrients that plants need. In a forest, where no one goes to add fertilisers, the decomposition of dead leaves, and other plant and animal matter enriches the soil with nutrients. As we discussed just above, bacterium like *Rhizobium*, also help in making the soil rich in nitrogen.



Keywords

autotrophic nutrition	mode of nutrition in which organisms prepare their own food.
chlorophyll	green pigment present in the leaves of plants.
heterotrophic nutrition	mode of nutrition in which organisms do not prepare their own food; they derive their food from plants, or animals, or both.
host	the living organisms from which a parasite derives its food.
insectivorous plants	insect-eating plants.
nutrition	the process, of obtaining, and utilising, food by a living organism.
parasitic nutrition	mode of nutrition in which non-green plants live on other living organisms and obtain their food from them.
photosynthesis	the process through which green plants prepare their own food.
saprotrophic nutrition	mode of nutrition in which some plants feed on dead and decaying matter.
stomata	tiny pores that are present on the surfaces of leaves; useful for exchange of gases.
vessels	channels, to transport water and minerals, to different parts of the plant.

You Must Know

1. The process of obtaining, and utilising, food is known as nutrition.
2. There are two types of nutrition—autotrophic and heterotrophic.
3. Autotrophic nutrition is the mode of nutrition in which green plants synthesise their own food by the process of photosynthesis.
4. Photosynthesis is the process by which green plants make their own food. The plants use simple chemical substances, like carbon dioxide, water and minerals, for synthesising their food, in the presence of sunlight/light.
5. During photosynthesis plants take in carbon dioxide and release oxygen; this released oxygen is utilised by living organisms for their survival.

6. Heterotrophic nutrition is the mode of nutrition used by some plants and practically all animals. It is used by all organisms that cannot synthesise their own food and depend on other sources for their food.
7. Heterotrophic nutrition has been sub-divided into three categories: saprotrophic, parasitic and holozoic nutrition.
8. Organisms, which derive nutrition from the body of other living organisms (host), are called parasites; for example, *cuscuta* (*amarbel*) and insect-eating plants.
9. Insect-eating plants are called insectivorous plants. Pitcher plant is an example of an insectivorous plant.
10. Saprotrophic nutrition is the process by which the organisms feed on dead and decaying matter.
11. In symbiotic relationship two organisms live in close association and develop a relationship that is beneficial to both.
12. The soil needs to be continuously replenished to remain productive. This is because the plants growing on it, and the small organisms living in it, keep on depriving it, of the nutrients present in it.

Something To Know

A. Fill in the blanks.

1. Animals are ______________ as they cannot synthesise their own food.
2. The ______________, of a plant, absorb water and minerals from the soil.
3. During photosynthesis plants take in ______________ and release ______________.
4. ______________ are the tiny pores through which leaves exchange gases.
5. Insect eating plants are called ______________ plants.
6. An essential raw material needed for the process of photosynthesis, and

   (a) available in the soil is ______________.

   (b) available in the air is ______________.

B. Match the following:

1.	Chlorophyll	(a) Autotrophs
2.	Lichens	(b) Saprotrophs
3.	Fungi	(c) Symbiotic relationship
4.	*Amarbel*	(d) Leaf
5.	Plants	(e) Parasite

C. Tick (✓) the correct option.

1. Green plants, that can synthesise their own food, are known as—

   ☐ heterotrophs ☐ parasites

   ☐ autotrophs ☐ saprotrophs

2. The food factory, of the plant, is its—

   ☐ root ☐ flower

   ☐ stem ☐ leaf

3. Which of the following is an insectivorous plant?

- [ ] pitcher plant
- [ ] leguminous plant
- [ ] green plant
- [ ] *amarbel*

4. Mushroom is an example of a/an—

- [ ] saprotroph
- [ ] parasite
- [ ] autotroph
- [ ] insectivorous

5. An organism, that fixes nitrogen in the soil, is—

- [ ] mushroom
- [ ] *mucor*
- [ ] *rhizobium*
- [ ] *cuscuta*

D. Answer the following questions in brief.

1. Why is nutrition important for a living organism?
2. How do green plants synthesise their food?
3. State the role of 'vessels' present in a plant.
4. Define the following terms:

   (a) Symbiotic relationship
   (b) Nutrients
   (c) Saprotrophic mode of nutrition
   (d) Photosynthesis

5. When some wheat dough was left in the open for a few days, it started emitting a foul smell. State, why?

E. Answer the following questions.

1. Why would life not be possible on the earth in the absence of photosynthesis?
2. Give reasons for the following:

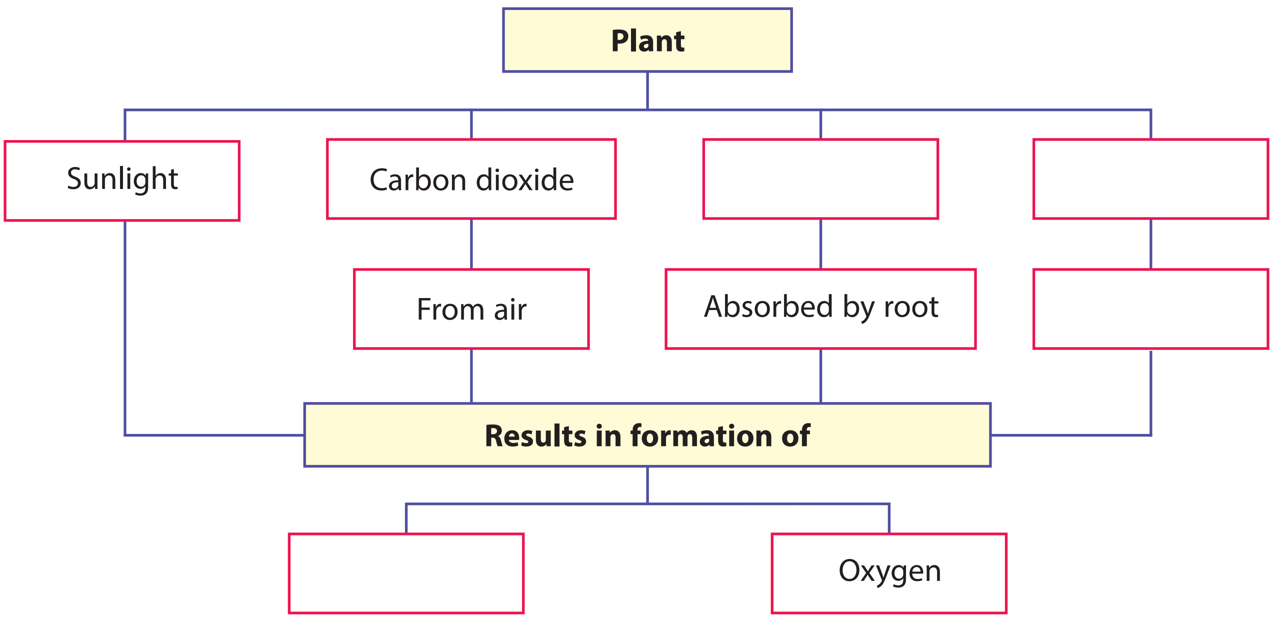
   (a) Mushroom is a saprotroph.
   (b) Sun is the ultimate source of energy for all living organisms.
   (c) The leaf of a plant 'dies out' if its stomata are blocked.
   (d) Leaf is known as the food factory of the plant.

(e) Lichen is a 'living partnership' between a fungus and an alga and this 'partnership' is beneficial to both.

3. Why do some plants feed on insects? How does a pitcher plant catch insects?

4. How do *rhizobium* bacteria and leguminous plants help each other in their survival?

5. Complete the web chart.



Value Based Question

The teacher told her students the story of the film *Dost*. She told them that, in that film, the friendship, between a visually challenged boy and a lame boy, helps them both to face, and overcome, the very many challenges of their day–to–day life. She went on to compare their friendship with the 'symbiotic relationship' between two organisms.

1. Suggest any two 'values' that, according to you, must have been there in the two friends of the film *Dost*.

2. In what way is the friendship, between the two boys, similar to the 'symbiotic relationship' between two organisms?

3. Give one example of a 'symbioitic relationship' between two organisms.

Something To Do

1. Compose a few lines/poem on the 'utility of plants'.

2. **Why is it important to increase the 'forest cover'?**

3. **Keep a stale, moist piece of bread in a warm corner of the kitchen and observe it for 3–4 days. Can you identify the organism growing on the piece of bread? Identify its mode of nutrition.**

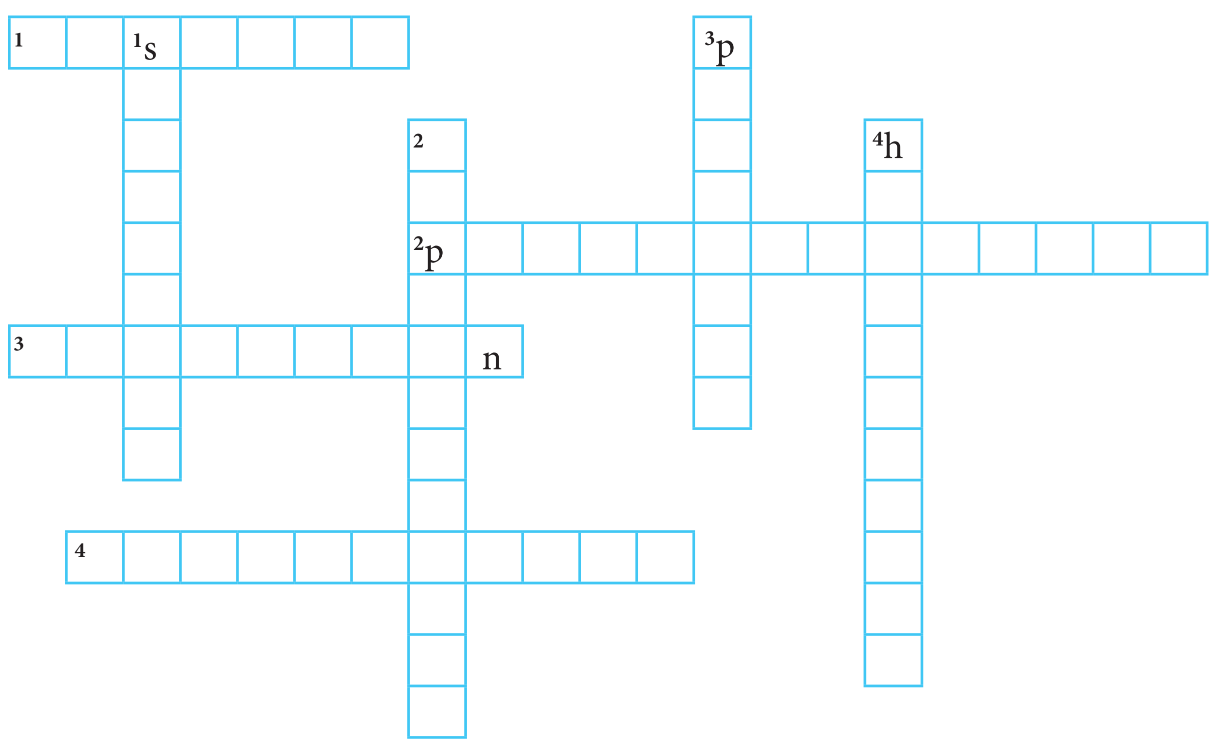
4. **Solve the crossword puzzle with the help of the clues given below.**

ACROSS →

1. A plant parasite.
2. The process by which green plants prepare their food.
3. The process of obtaining, and utilising, food.
4. Green pigment present in the leaves of plants.

DOWN ↓

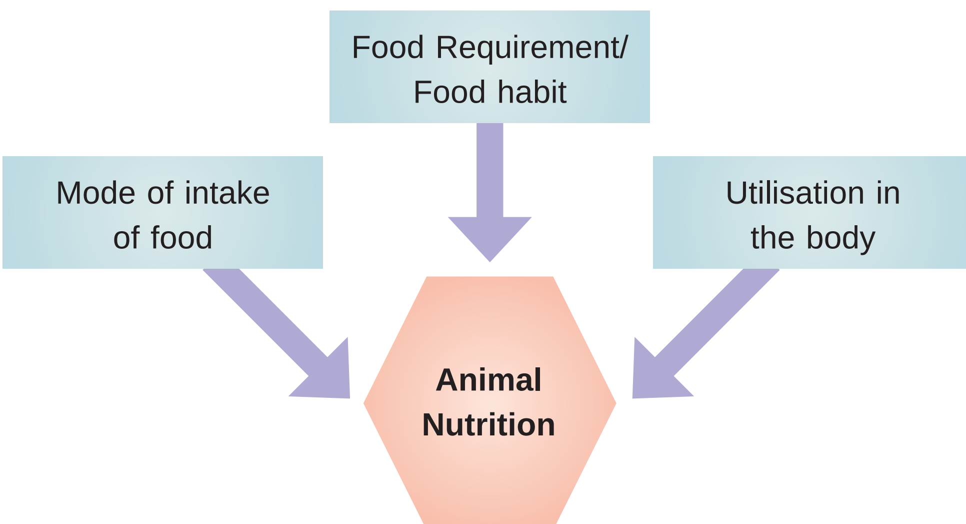
1. Two different organisms that live together and thereby, benefit from each other.
2. Organism feeding on dead matter.
3. An organism deriving food from another living organism.
4. Organism that cannot prepare its own food.



CHAPTER

2 Nutrition in Living Organisms—Animals and Man

You have already learnt in Chapter 1 that plants can prepare their own food and are called **autotrophs**. Animals cannot prepare their own food and are called **heterotrophs**. Animals eat complex food materials but break it down into simpler forms in their bodies. Their body gets the required nutrition through the three main steps shown below—



Three main steps in Animal Nutrition

Modes of Intake of Food

The method of taking in food is different in different organisms. The relevant parts of their body get modified in a manner that makes it easy for them to eat their food. A sparrow has a short beak to pick up seeds and worms. The long, tubular beak, of the humming bird, helps it to suck nectar from the flowers. The cow has sharp incisors and flat molar teeth that help it to cut and grind plant materials. The jaws of many snakes enable them to swallow animals that may be much larger than the size of their head.

Sparrow

Humming bird

Cow

Snake

Mode of intake of food differs in different organisms

Food Habits of Animals

On the basis of their food habits, animals have been categorised into three different categories.

The animals, like cow and deer, that eat only plant materials, are called **herbivores**.

The animals, like lion and tiger, that eat only other animals, are called **carnivores**.

Animals, like bear and human beings, that eat both animals and plant materials, are called **omnivores**.

Modes of Nutrition

It is their food that provides animals their required nutrition. As we have mentioned in the previous chapter, the nutrition requirements of heterotrophs, i.e. heterotrophic nutrition, are met by them in three different ways. These three modes of nutrition are:

- Saprotrophic nutrition
- Parasitic nutrition
- Holozoic nutrition

Saprotrophic Nutrition

We have already learnt in the previous chapter that, the mode of nutrition, in which an organism obtains its (required) nutrients, from dead and decaying plant and animal matter, is known as **saprotrophic nutrition**. Such an organism secretes enzymes outside, digests the organic food and absorbs the soluble organic compounds.

Most fungi and some bacteria are saprotrophs.

Parasitic Nutrition

The mode of nutrition, in which an organism (known as a parasite) obtains food from some other living organism (known as the host), of a different group, is known as **parasitic nutrition**. Parasites may live on, or in, the body of another living organism. In this mode of nutrition the parasite is benefitted while the host gets harmed.

Roundworms, head louse, body louse and tapeworm are parasites.

Holozoic Nutrition

This is a mode of nutrition, in which organisms, like *Amoeba* and human beings, eat food that may be in solid or in liquid state. This food is taken into the body (or eaten), and then it is broken down (or digested) to provide the required nutrition to the body.

Having understood the three different modes of nutrition, used by heterotrophs, let us now talk, in detail, how the human body gets its required nutritions.

Nutrition in Humans

The food that we eat passes though a long muscular tube (called the alimentary canal) present inside our body. This canal begins at the mouth and ends at the anus. The food is broken down into tiny molecules that are carried, by blood, to all parts of the body. The sequence of steps, involved in this process, are as follows:

- ### Ingestion

  The act of getting, and eating, food is called **ingestion**. In humans, it takes place through the mouth where the teeth help in chewing the food.

- ## Digestion

  The process of breakdown, of complex molecules into simple soluble ones, is called **digestion**. This digestion of food gets done, with the help of certain chemicals, called the **enzymes**. The process of digestion starts in the buccal cavity and gets completed in the small intestine.

- ## Absorption

  The digested food is absorbed by the walls of the small intestine from where it gets passed on to the blood.

- ## Assimilation

  The absorbed food is utilised by the body, for growth and formation, of body parts. This process is known as **assimilation**.

- ## Egestion

  The elimination, of undigested food, from the alimentary canal, is known as **egestion**.

  The alimentary canal is made up of the following (body) parts:

  (a) Buccal cavity

  (b) Food pipe (or Oesophagus)

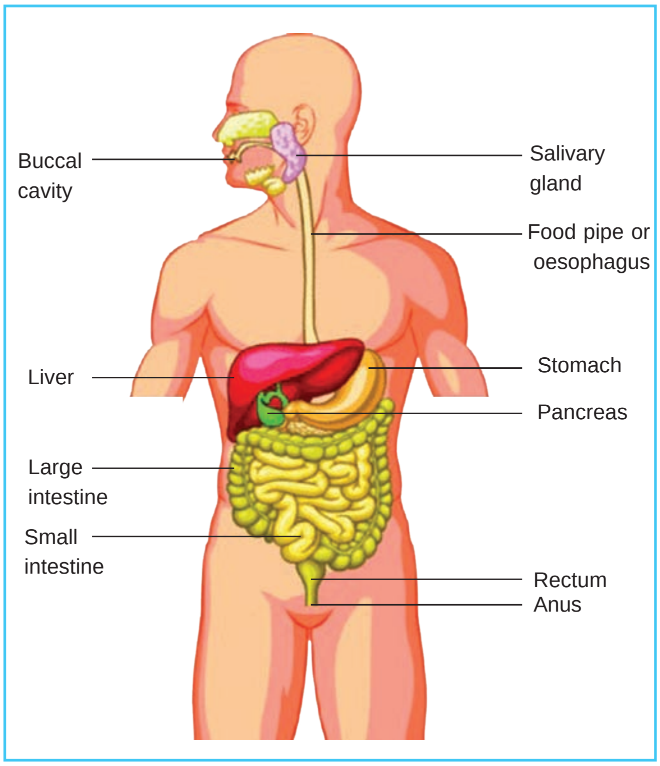
  (c) Stomach

  (d) Small Intestine

  (e) Large Intestine

  (f) Rectum

  (g) Anus



Human Digestive System

Besides these parts, there are a number of glands, associated with the alimentary canal, which play their roles in the process of digestion of food. The salivary glands, pancreas and the liver, are the three main such glands.

The Alimentary canal, along with these associated glands, form the overall **digestive system**. We now look at the specific function/role of the different parts of the digestive system.

Mouth and Buccal Cavity

In humans, the food is taken in through the mouth from where it goes into the buccal cavity. Our buccal cavaity contains the teeth and the tongue. The Salivary glands, present here, release saliva into the buccal cavity.

Activity 1

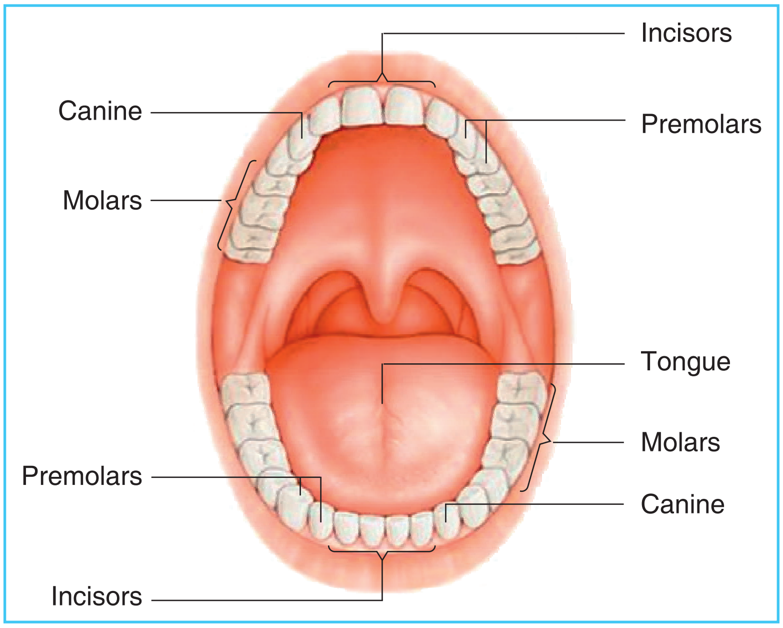
Chew a piece of bread for 3-4 minutes. Note the change in taste as you chew it.

Describe the observed change in taste. Why is there a change in taste?

The saliva contains digestive juices that break down starch to form sugar. Hence, a starchy substance (that is tasteless), when chewed for sometime, tastes sweet.

The Teeth

Teeth, are rooted in the sockets of the bones of the jaw. These are covered by a white, strong, shining, protective material, called the **enamel**. Teeth help in cutting, tearing and grinding of food.



A view of buccal cavity

Adult humans typically have 32 teeth—16 in upper jaw and 16 in the lower jaw—that fit together and help them to chew food. Humans develop two sets of teeth during their life. The first set of teeth are 20 small teeth, also known as **baby teeth** or **milk teeth**. They start appearing, above the gumline, when a baby is six, or seven, months old. By the time a child is (around) six years old, a second set of 32 larger teeth, called **permanent teeth,** 'come out' from the gums and (eventually) replace the milk teeth.

Humans have four different types of teeth that perform different functions.

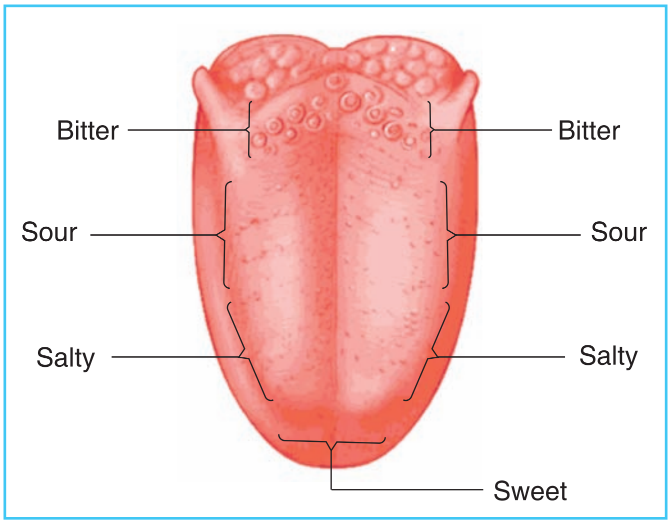
- **Incisors** are used for cutting of food.
- **Canines** are used for tearing of food.
- **Premolars** are used for grinding of food.
- **Molars** are also used for grinding of food.

Do You Know?

Enamel is the hardest substance in the human body and covers the outer portion of the teeth. It is made up of mineral salts (of calcium and magnesium) and keratin (a protein). It can withstand quite high pressures.

The Tongue

Tongue is a muscular organ attached to the floor of the buccal cavity. It is free to move at its front end where it can move in all directions. The tongue helps in mixing up of saliva with food; it also helps in swallowing food. It has four types of taste buds which help us to know about the sweet, sour, bitter and salty tastes, associated with different types of items, in our food.



Taste buds on tongue

Do You Know?

Tongue is a busy organ. The tongue serves as an organ of taste, with taste buds scattered over its surface. During chewing, the tongue holds the food against the teeth; in swallowing, it moves the food back into the buccal cavity, and then into the oesophagus (when the pressure of the tongue closes the opening of the trachea, or windpipe).

Activity 2

Take some water that has been obtained after boiling rice. This water has some starch dissolved in it. Add 2-3 drops of this water to four glass bowls (or small transparent *katories*, or glasses), and label them as A, B, C and D. Add a teaspoon of plain water to each glass bowl. Now add a teaspoon of your saliva to bowl B. Heat another teaspoon of your saliva, over a flame, for about 20 seconds and add this to bowl C. After about 10 minutes, add 4-5 drops of iodine in each bowl, except bowl D. Tabulate your observations in the following table.

Glass Bowl	Quantity of water with starch	Quantity of saliva	Colour change observed after 10 minutes
A			
B			
C			
D			

Presence of blue-black colour, after the addition of iodine drops, shows the presence of starch. What happened to the starch in bowl B? Why did the blue-black colour not appear in bowl B and bowl D?

For the Teacher

The blue-black colour does not appear in bowl B because the saliva 'breaks' the starch into simpler sugars which do not undergo a colour change with iodine.

There is, however, a colour change in bowl C. This is because the saliva loses it property of 'breaking' starch after heating. The starch, therefore, stays as such. Hence, the addition of iodine causes a colour change.

Note: While doing this activity, teacher must ensure hygiene.

The Food Pipe (or Oesophagus)

The **food pipe** is a long, narrow and muscular tube that connects the buccal cavity to the stomach. Food, that has been chewed in the mouth, is pushed downward into the oesophagus. From here, the onward movement of the food is due to the movement of the muscles, present in the wall of the oesophagus.

Do You Know?

Epiglottis is a flap-like structure present at the top of the wind pipe. It closes the wind pipe when we swallow food and prevents the food from entering our lungs.

The Stomach

The **stomach** is the widest part of the alimentary canal. It is a thick-walled, sac-like muscular organ. It receives food, from the oesophagus, and passes it into the small intestine. The inner lining of the stomach secretes gastric juices, which have mucus, hydrochloric acid and enzymes present in them.

Mucus protects the inner lining of the stomach. Hydrochloric acid kills bacteria. It also provides the acidic medium, needed for digestion of food, by the enzymes in the stomach. The enzymes, in the stomach, break down proteins to simpler substances, like amino acids.

Do You Know?

Stomach with a hole

On June 6, 1822, a person named Alexis St. Martin, was accidentally shot in the stomach. Dr. Beaumont treated his wound. Despite his best efforts, Dr. Beaumont could not close the hole in his stomach that never fully healed.

Dr. Beaumont recognised that he had, in St. Martin, the unique opportunity to observe digestive processes. He began to perform experiments, on digestion, in the stomach of St. Martin. Most of these experiments were conducted by tying a piece of food to a string, and inserting it, through the hole, into St. Martin's stomach. Every few hours, Dr. Beaumont would remove the food and observe how well it had been digested. Dr. Beaumont also extracted a sample of gastric acid (digestive juice) from St. Martin's stomach for analysis. He also used samples of this stomach acid to "digest" bits of food in cups. This led to the important discovery that the stomach acids help to digest the food (into simple and soluble) nutrients, that the stomach can use. It was, thus, realised that digestion is primarily a chemical process, and not a mechanical one.

The Small Intestine

The small intestine is not all that small. It is about 6–7 metres long. It helps in digestion by using three types of secretions.

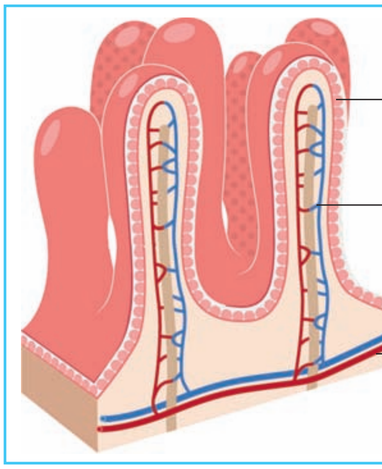
1. Secretions from liver—The Liver is the largest gland in the human body. It is present slightly below the stomach, on the right side. It secretes bile juice, that is stored in a bag-like structure, called the **gall bladder**. This bile juice plays an important role in digestion of fats.

2. Secretions from pancreas—**Pancreas** is a yellow, leaf-shaped, gland, located just below the stomach. It secretes pancreatic juice; this juice acts on carbohydrates, proteins and fats and breaks them into simpler forms.

3. Secretions from the small intestine—The inner wall, of the small intestine, itself secretes the intestinal juice. This juice digests carbohydrates, proteins and fats.

The small intestine, uses the bile, pancreatic and intestinal juices to complete the process of digestion of food in itself. Here (i) carbohydrates are digested to simple sugars like glucose (ii) proteins are broken down to amino acids and (iii) fats are broken down into fatty acids and glycerol.

Absorption of digested food in the small intestine



Parts of small intestine showing villi

The inner wall, of the small intestine, absorbs the digested food. It has a large number of finger-like projections, called **villi**. The villi increase the effective surface area for absorption of digested food. This absorbed food is passed to blood vessels, present in the villi. The 'food', thus, get transported to all parts of the body via the blood. It is used to produce energy and to build complex substances required by the body. This whole process is called **assimilation**.

The Large Intestine

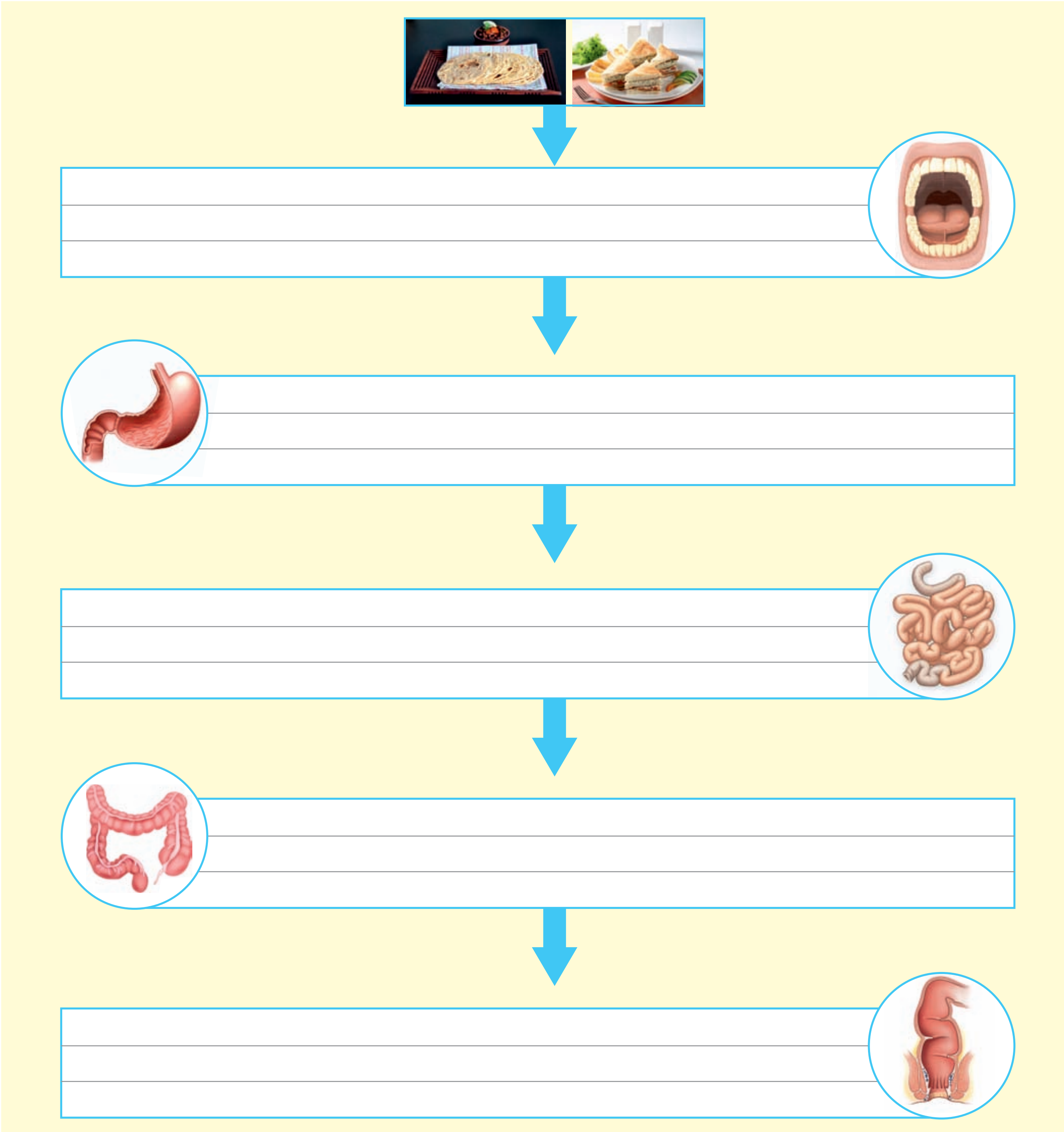
Large intestine is wider and shorter than the small intestine. It is about 1.5 metres in length. The undigested, and unabsorbed, food enters the large intestine. Here, the excess of water and some minerals are absorbed from the undigested food. The left over waste part of food passes to the rectum and is stored there as faecal matter. The faeces are eliminated through the anus. This process is called **egestion**.

Activity 3

Tracing the journey of a *chappati*/sandwich.

We all eat a *chappati*, or a sandwich, quite often. Our digestive system 'breaks down' this food item into simpler forms. It, thus, helps it to provide the energy and nutrients our body needs for its maintainence and growth.

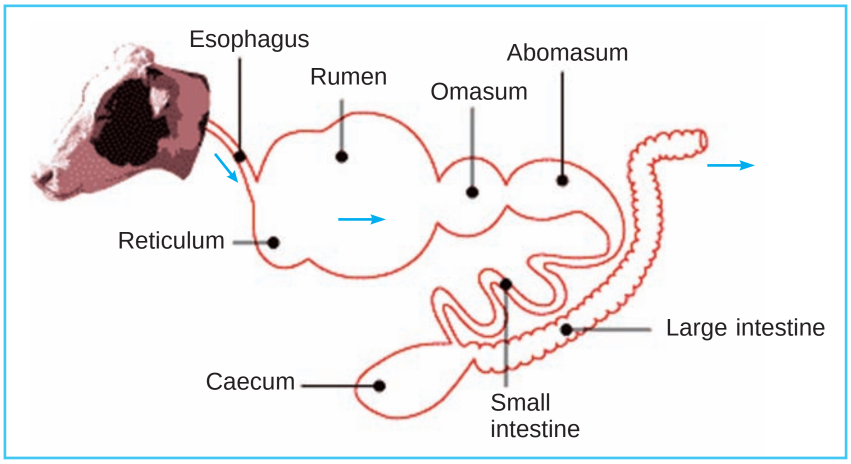
Use the flow chart, given on the next page, to trace the journey of a *chappati*/sandwich to show how it goes from just being a 'food item on your plate' to 'energy for life'. Write just one/two sentence/s, in the space provided, to highlight the role of, each of the indicated parts, in this eventful journey.



Nutrition in Cud Chewing Herbivore Animals

Cud chewing herbivore animals are called **ruminants.** Cow, deer, camel, buffalo, sheep and giraffe are some of the well-known **ruminants.** They have a special four-chambered stomach.

The first chamber is the largest and is called **rumen**. These animals first swallow the food quickly and store it in their rumen. The rumen has some micro-organisms that help in partial digestion of the cellulose of the plant materials. This food is now called **cud**. The ruminants, later on, bring this cud back to their mouth and chew it thoroughly. This process is called **rumination**. The thorough chewing of food during rumination, helps to break down the rich cellulose content of the plant materials. This 'breaking down' makes it easy, for these animals, to digest the cellulose content.



Digestive system of a cow

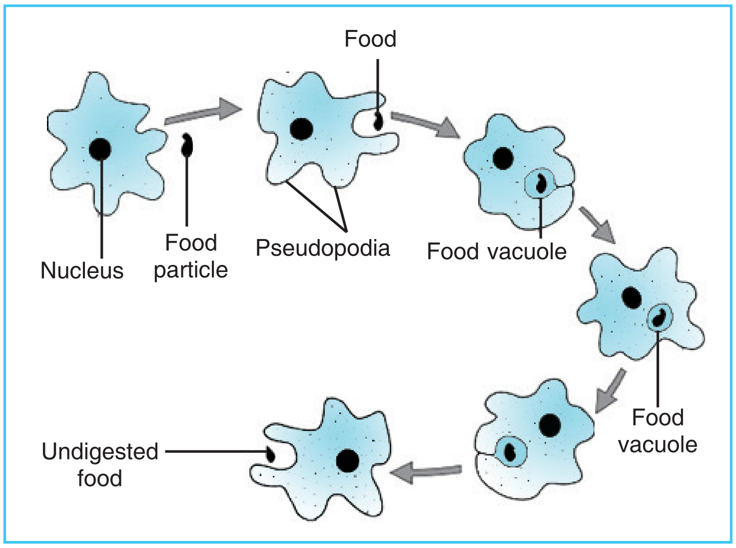
Ruminants also have a spacious bag-like structure, between their small intestine and the large intestine. This is called **caecum**. The bacteria, present in the caecum, help in further digestion of the cellulose of the food.

It is interesting to note here that such bacteria are not present in the human digestive system. Human beings cannot, therefore, digest the cellulose, which goes on to form roughage. Roughage helps in the bowel movement in the human body.

Nutrition in Amoeba

Amoeba is a microscopic, unicellular, organism found in moist soil, ponds and lakes. It is surrounded by a cell membrane. It constantly changes its shape and moves with the help of pseudopodia (*pseudo* = false, *podia* = feet). Pseudopodia also help it in capturing food.

Amoeba feeds on small microscopic organisms like bacteria and algae. When *Amoeba* comes in contact with food, it produces pseudopodia



Nutrition in Amoeba

around the food particle. As the cell membranes of the pseudopodia fuse, the food gets trapped in a food vacuole.

Digestive juices are secreted into this vacuole to digest the food. The digested food is absorbed and used for production of energy, movement and maintenance of the organism. The undigested food, present in the food vacuole, is expelled, from the *Amoeba*, by the process of **egestion**.

Keywords

absorption	process by which digested food passes into the blood.
alimentary canal	the long muscular tube in the human body through which food passes after its ingestion.
assimilation	process of using the absorbed food for growth and for producing energy.
buccal cavity	oral cavity located at the upper end of the alimentary canal.
caecum	bag-shaped part, at the beginning of the large intestine, present in ruminants.
canines	pointed teeth used for tearing of food.
digestion	breaking down of complex food into simple soluble forms with the help of digestive juices.
enamel	white substance that covers the teeth.
egestion	process of elimination of undigested food.
ingestion	process of 'taking in' of food.
incisors	front teeth used for cutting and biting.
molars	last teeth, that are larger and flat, and are used for crushing and grinding food.
oesophagus	food pipe.
premolars	teeth situated next to canines, and used for crushing and grinding food.
pseudopodia	false feet of amoeba which it uses to (i) trap its food and (ii) for its movement.
ruminants	cud chewing herbivore animals.
villi	small projections, in the inner walls of the small intestine of human beings; these help in absorption of the digested food.

You Must Know

1. Animal nutrition includes nutrient requirement, mode of intake of food and its utilisation in the body.
2. Heterotrophic nutrition is of three types—saprotrophic nutrition, parasitic nutrition and holozoic nutrition.
3. The process of nutrition in animals involves ingestion, digestion, absorption, assimilation and egestion.
4. The human digestive system consists of the mouth, oesophagus, stomach, small intestine and large intestine; ending in the rectum and anus.
5. The main digestive glands, which secrete digestive juices, are the salivary glands, the liver and the pancreas. The stomach wall and the wall of the small intestine also secrete digestive juices.
6. The different types of teeth, in humans, are—incisors, canines, premolars and molars.
7. Digestion begins in the buccal cavity and continues in the stomach and small intestine. The digested food gets absorbed, in the blood vessels, from the small intestine.
8. The absorbed substances are transported to different parts of the body. Water, and some salts, are absorbed from the undigested food in the large intestine.
9. The undigested, and unabsorbed residue, are expelled out of the body as faecal matter through the anus.
10. Cud chewing herbivore animals are called ruminants. They first, quickly swallow food and store it in their rumen. Later on, the food returns to the mouth and the animals chew it on thoroughly.
11. Amoeba ingests its food, with the help of its false feet, called pseudopodia. The food is digested in the food vacuole.

Something To Know

A. Fill in the blanks.

1. The digestion of food in humans starts in the ______________ and is completed in the ______________.
2. ______________, present in the stomach, kills bacteria.
3. The largest gland in the human body is the ______________.
4. Partially digested food, that is chewed again by grass eating animals, is called the ______________.
5. *Amoeba* uses ____________________ for locomotion and for capturing its food.

B. Match the following:

1. Gall bladder	(a) Bile Juice
2. Proteins	(b) Cow
3. Intestinal wall	(c) Absorption
4. Rumen	(d) False feet
5. Pseudopodia	(e) Amino acids

C. Tick (✓) the correct option.

1. Organisms, that can synthesise their own food, are called—

   ☐ heterotrophs ☐ parasites

   ☐ autotrophs ☐ saprotrophs

2. Cow is a/an—

   ☐ saprotroph ☐ parasite

   ☐ autotroph ☐ heterotroph

3. Animals, that eat both plant materials and animals, are called—

- [ ] herbivores
- [ ] omnivores
- [ ] carnivores
- [ ] ruminants

4. Which one of these is not a part of the alimentary canal?

- [ ] stomach
- [ ] anus
- [ ] liver
- [ ] large intestine

5. Bile juice is released by the—

- [ ] salivary glands
- [ ] pancreas
- [ ] liver
- [ ] large intestine

D. Answer the following questions in brief.

1. Define the following terms:

   (a) Holozoic nutrition

   (b) Alimentary canal

2. Give the meaning of the terms:

   (a) Assimilation

   (b) Rumination

3. Name the organs that make up the human alimentary canal.

4. State two differences between milk teeth and permanent teeth.

5. Name the four types of teeth in the human mouth.

6. State the function of the (a) incisor teeth (b) premolar teeth.

7. State the role of acid in the human stomach.

8. State the function of (a) bile juice and (b) pancreatic juice in the human digestive system.

E. Answer the following questions.

1. Draw a neat, well labelled diagram of the human digestive system.
2. Justify the following statements:
   (a) Crow is an omnivore.
   (b) It is said that the mode of nutrition, in human beings and *Amoeba*, is quite similar.
3. Give reasons for the following:
   (a) Ingestion of food is difficult without teeth.
   (b) If we chew rice, or bread, for a few minutes, it starts tasting sweet.
   (c) Bacteria are present in the caecum of ruminants.
4. Explain how digested food gets absorbed into the blood.
5. State, in one/two sentence/s each, the various processes involved in nutrition in ruminant animals.
6. Explain ingestion of food, in *amoeba*, through a diagram.

Value Based Question

The biology teacher, who was also the coach of the school cricket team, would often compare his team members with the different 'organs' of their digestive system. He would tell them to concentrate on their respective roles and to work as a team in a selfless and dedicated way. This, he would say, would enable them to succeed in winning matches in the same way as the 'team', of the organs of the digestive system, 'succeeds' in digesting, and using, the 'ingested food'.

1. State two of the values that the teacher wanted his students to 'have in them'.
2. Try to make a list of 'eleven names' that are a part of the 'team' that makes up the human digestive system.
3. Have a group discussion on the importance of 'Team work' in day-to-day life.

Something To Do

1. Make a PowerPoint presentation on the various ways in which animals ingest their food. For example—
   [Herbivores–Cow; Carnivores–Lion; Blood suckers–Leech; Fruit eating–birds; Fluid feeders–Butterflies, moth, earthworm; Insectivore–Frog]
2. Collect data, from the parents of your five classmates, about their milk teeth. Tabulate your data as given below.

S.No.	Name	Age at which first tooth fell	Age at which last tooth fell	No. of teeth lost	No. of teeth replaced

   Use the collected data, to estimate the average age at which children lose their milk teeth.
3. Make a model of the digestive system (using clay/plasticine to make the organs) and rubber pipes/ribbons to make the food pipe and small intestine.
4. Activity—Assign a particular organ of the digestive system to each student and ask them to enact the role of it. The students need to follow the given guidelines. They should introduce themselves as a particular organ, explain its structure and function, its importance and significance in the human body and name some diseases, associated with the 'organs', represented by them.
   The teacher can judge their role play by considering their presentation, content, visual aid used, clarity of the concepts, etc.