CHAPTER 1

SQUARES AND SQUARE ROOTS

INTRODUCTION

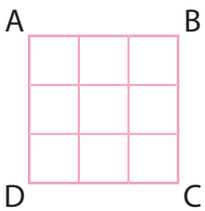
In previous classes, we have already studied the squares of many natural numbers.

For example, $3^2 = 3 \times 3$

We say that 3 to the power 2 or 3 squared is 9.

$$3^2 = 3 \times 3 = 9$$

Now, let us take a square figure ABCD in order to explain the given example. Here, each side of the square has 3 units.



$\therefore$ Area $= 3 \times 3 = 3^2$ square units

$= 9$ square units

In this Chapter, we shall be concentrating on the procedures to find the positive square roots of positive rational numbers.

SQUARES

Look at the examples given below:

$2 \times 2 = 4 = 2^2$

$3 \times 3 = 9 = 3^2$

$4 \times 4 = 16 = 4^2$

Similarly, $a \times a = a^2$

So, we conclude that—

The square of a number is the product obtained by multiplying the number by itself.

Numbers, such as 1, 4, 9, 16, 25, 36 are called **perfect squares**.

Remember

A given number is called a **perfect square** or a **square number** if it is the square of some natural number. These numbers are exact squares and do not involve any decimals or fractions.

To find out whether a given number is a perfect square or not, write the number as a product of its prime factors. If these factors exist in pairs, the number is a perfect square.

Let us take an example to find whether the given number is a perfect square or not.

Example 1: Which of the following numbers are perfect squares?

(i) 256 (ii) 154 (iii) 720

Solution: (i) **Step 1:** $256 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}$

Step 2: Prime factors of 256 can be grouped into pairs and no factor is left out.

$\Rightarrow 256 = (2 \times 2 \times 2 \times 2)^2 = (16)^2$

$\therefore$ 256 is a perfect square of 16.

(ii) **Step 1:** $154 = 2 \times 7 \times 11$

Step 2: No prime factor exists in pairs.

$\therefore$ 154 is not a perfect square.

(iii) **Step 1:** $720 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times 5$

Step 2: In prime factors of 720, factor 5 is left ungrouped.

$\therefore$ 720 is not a perfect square.

Facts About Perfect Squares

(i) A number ending with an odd number of zeroes (one zero, three zeroes and so on) is never a perfect square,

e.g. 150, 25000, 350 are not perfect squares.

(ii) Squares of even numbers are always even,

e.g. $8^2 = 64$ $12^2 = 144$ $20^2 = 400$

(iii) Squares of odd numbers are always odd,

e.g. $7^2 = 49$ $13^2 = 169$ $21^2 = 441$

(iv) The numbers ending with 2, 3, 7, 8 are not perfect squares,

e.g. 32, 243, 37, 368 are not perfect squares.

(v) The square of a number other than 0 and 1, is either a multiple of 3 or exceeds the multiple of 3 by 1.

- Examples of multiples of 3.

  $3^2 = 9$ $\qquad$ $12^2 = 144$

- Examples of multiples of 3 exceeded by 1.

  $4^2 = 16 = (15 + 1)$ $\qquad$ $13^2 = 169 = (168 + 1)$

(vi) The square of a number other than 0 and 1, is either a multiple of 4 or exceeds a multiple of 4 by 1.

- Examples of multiples of 4.

  $6^2 = 36$ $\qquad$ $8^2 = 64$

- Examples of multiples of 4 exceeded by 1.

  $7^2 = 49 = (48 + 1)$ $\qquad$ $9^2 = 81 = (80 + 1)$

(vii) The difference between the squares of two consecutive natural numbers is equal to their sum.

Let us take two consecutive natural numbers, 3 and 4.

$4^2 - 3^2 = 16 - 9 = 7 = 4 + 3$

Thus, in general, if n and $(n + 1)$ be two consecutive natural numbers,

$$\text{then } (n+1)^2 - n^2 = \left[(n+1)\,(n+1)\right] - n^2$$
$$= n^2 + n + n + 1 - n^2 = n + (n+1)$$

(viii) The square of a natural number n is equal to the sum of the first n odd natural numbers,

e.g. $1^2 = 1 = $ sum of the first one odd natural number

$2^2 = 1 + 3 = $ sum of the first two odd natural numbers

$3^2 = 1 + 3 + 5 = $ sum of the first three odd natural numbers

$4^2 = 1 + 3 + 5 + 7 = $ sum of the first four odd natural numbers

and so on.

(ix) Squares of natural numbers composed of only digit 1, follow a peculiar pattern.

$1^2 = 1$

$11^2 = 121$

$111^2 = 12321$

$1111^2 = 1234321$

$11111^2 = 123454321$

We can also observe that the sum of the digits of every such number is a perfect square 1, 121, 12321, 1234321.

$$1 = 1 = 1^2$$
$$1 + 2 + 1 = 4 = 2^2$$
$$1 + 2 + 3 + 2 + 1 = 9 = 3^2$$
$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16 = 4^2$$
$$1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25 = 5^2$$

See, how beautiful patterns of numbers are made above.

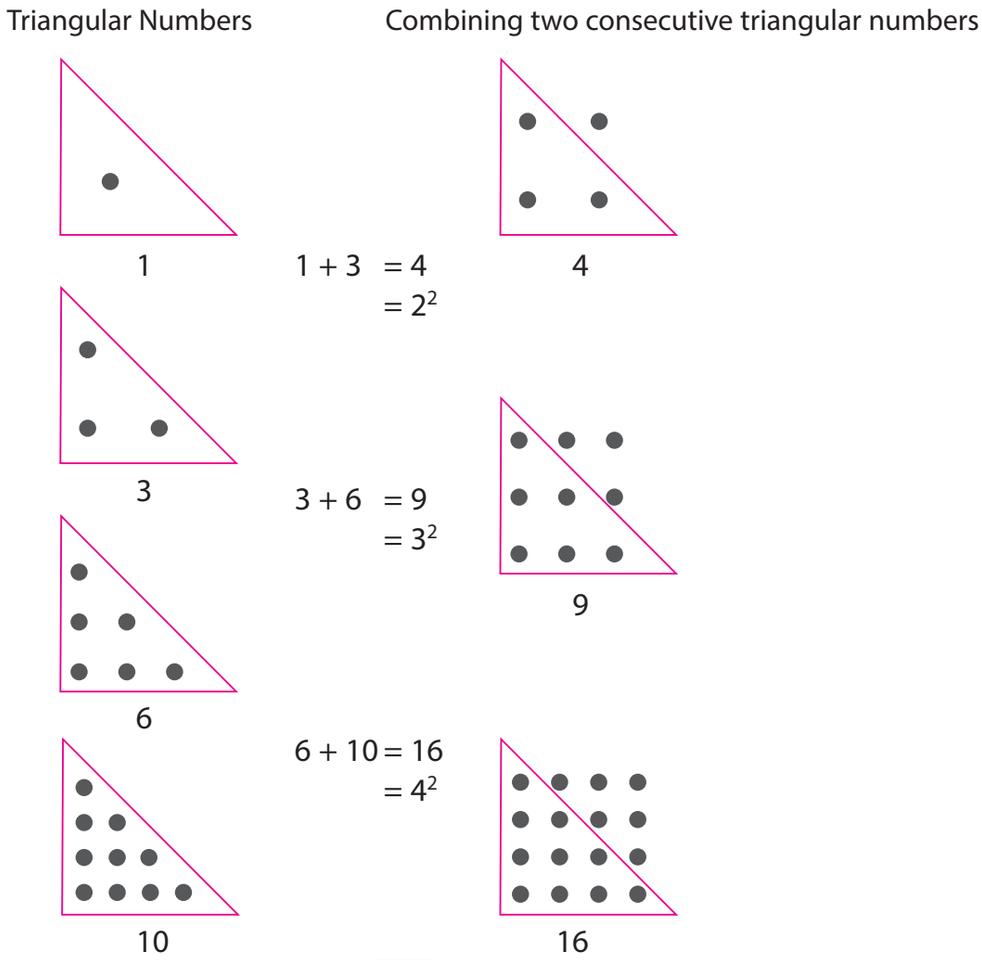
Some Interesting Patterns

▶ Adding triangular numbers

Remember

Numbers whose dot patterns can be arranged as triangles are called **triangular numbers**.

Let us add triangular numbers.



Observe the following pattern and fill in the blanks.

$$1 + 3 = 2^2$$
$$1 + 3 + 5 = 3^2$$
$$1 + 3 + 5 + 7 = \_\_\_\_\_$$
$$1 + 3 + 5 + 7 + 9 = \_\_\_\_\_$$
$$1 + 3 + 5 + 7 + 9 + 11 = \_\_\_\_\_$$
$$1 + 3 + 5 + 7 + 9 + 11 + 13 = \_\_\_\_\_$$

Numbers between square numbers

Let us observe some interesting patterns between two consecutive square numbers.

We have $1^2 = 1$

$2^2 = 4$

The non-square numbers between 1 and 4 are 2, 3.

1, 2, 3, 4 ⟶ **2 non-square numbers**

The non-square numbers between 4 (= 2^2) and 9 (=3^2) are 5, 6, 7, 8.

4, 5, 6, 7, 8, 9 ⟶ **4 non-square numbers**

Now, let us put our observations in a tabular form.

Consecutive square numbers	Non-square numbers	Number of non-square numbers
1 and 4	2, 3	2
4 and 9	5, 6, 7, 8	4
9 and 16	10, 11, 12, 13, 14, 15	6
16 and 25	17, 18, ... 24	8
25 and 36	26, 27, ... 35	10

and so on.

Now, let us generalise our observations.

Between 1^2 (= 1) and 2^2 (= 4), there are two non-square numbers (i.e. 2 *x* 1).

Between 2^2 (= 4) and 3^2 (= 9), there are four non-square numbers (i.e. 2 *x* 2).

Between 3^2 (= 9) and 4^2 (= 16), there are six non-square numbers (i.e. 2 *x* 3).

Between 4^2 (= 16) and 5^2 (= 25), there are eight non-square numbers (i.e. 2 *x* 4).

Between 5^2 (= 25) and 6^2 (= 36), there are ten non-square numbers (i.e. 2 *x* 5).

Can you say how many non-square numbers are there between 6^2 and 7^2?

We find that if we take any natural number, n and $(n + 1)$, the number of non-square numbers between n^2 and $(n + 1)^2$ is $2n$.

There are $2n$ non-perfect square numbers between the square of the numbers, n and $(n+1)$.

Worksheet 1

1. Which of the following numbers are perfect squares?

11, 16, 32, 36, 50, 64, 75

2. Which of the following numbers are perfect squares of even numbers?

121, 225, 784, 841, 576, 6561

3. Which of the following numbers are perfect squares?

100, 205000, 3610000, 212300000

4. By just observing the digits at ones place, tell which of the following can be perfect squares?

1026, 1022, 1024, 1027

5. How many non-square numbers lie between the following pairs of numbers?

(i) 7^2 and 8^2 (ii) 10^2 and 11^2 (iii) 40^2 and 41^2

(iv) 80^2 and 81^2 (v) 101^2 and 102^2 (vi) 205^2 and 206^2

6. Write down the correct number in the box.

(i) $100^2 - 99^2$ = ☐ = ☐

(ii) $27^2 - 26^2$ = ☐ = ☐

(iii) $569^2 - 568^2$ = ☐ = ☐

7. Observe the pattern in the following and find the missing numbers.

$$1\underline{2}1 = \frac{(22)^2}{1+2+1}$$

$$12\underline{3}21 = \frac{(333)^2}{1+2+3+2+1}$$

$$123\underline{4}321 = \_\_\_\_\_$$

$$12345\underline{4}321 = \_\_\_\_\_$$

$$1234\underline{5}4321 = \_\_\_\_\_$$

8. Which of the following triplets are Pythagorean?

(3, 4, 5), (6, 7, 8), (10, 24, 26), (2, 3, 4)

[**Hint** : Let the smallest even number be $2m$ and find m from it. Then, find $(2m, m^2 - 1, m^2 + 1)$. If you get the triplet, it is Pythagorean.]

Another way of finding a Pythagorean triplet is:

If 'a', 'b' and 'c' are three natural numbers with 'a' as the smallest of them, then,

(i) If 'a' is odd, sum of other two numbers is a^2 and their difference is 1.

(ii) If 'a' is even, sum of other two numbers is $\frac{a^2}{2}$ and their difference is 2.

SQUARE ROOTS

We know that

$$4^2 = 4 \times 4 = 16$$

We say square root of 16 is 4.

This is written as $\sqrt{16} = 4$.

Note: $(-4)^2 = 16$ Therefore, square root of 16 can be –4 also, but here we are taking only positive square root.

Let us see some more examples.

$$7^2 = 49 \longrightarrow \sqrt{49} = 7$$

$$5^2 = 25 \longrightarrow \sqrt{25} = 5$$

$8^2 = 64 \longrightarrow \sqrt{64} = 8$

In general, if $m^2 = n$ then $\sqrt{n} = m$

Hence, square root of a given natural number n is that natural number m whose square is n.

From the above examples, we observe that—

(i) the square root of an even number is even.

(ii) the square root of an odd number is odd.

(iii) the symbol for the square root is $\sqrt{\ }$.

(iv) if a is the square root of b, then b is the square of a.

Observe the following pattern.

$1 + 3 = 2^2$

$1 + 3 + 5 = 3^2$

$1 + 3 + 5 + 7 = 4^2$

$1 + 3 + 5 + 7 + 9 + 11 + 13 = 7^2$

$1 + 3 + 5 + 7 +$ ..... up to n terms $= n^2$

The sum of first n odd numbers is n^2.

Finding Square Root of a Number by the Repeated Subtraction Method

Let us find $\sqrt{9}$.

Step 1: $9 - 1 = 8$ → First odd number

Step 2: $8 - 3 = 5$ → Second odd number

Step 3: $5 - 5 = 0$ → Third odd number

We have subtracted from 9, the successive odd numbers 1, 3 and 5, and obtained 0 in **Step 3**.

$\therefore \quad \sqrt{9} = 3$

Consider another example.

Example 2: Find $\sqrt{121}$ by repeated subtraction method.

Solution : **Step 1:** $121 - 1 = 120$ **Step 3:** $117 - 5 = 112$

Step 2: $120 - 3 = 117$ **Step 4:** $112 - 7 = 105$

Step 5: 105 – 9 = 96
Step 6: 96 – 11 = 85
Step 7: 85 – 13 = 72
Step 8: 72 – 15 = 57
Step 9: 57 – 17 = 40
Step 10: 40 – 19 = 21
Step 11: 21 – 21 = 0

We have subtracted from 121, the successive odd numbers from 1 to 21, and obtained 0 in **Step 11**.

$\therefore \quad \sqrt{121} = 11$

Worksheet 2

Find the square root of the following numbers by the repeated subtraction method.

(i) 16 (ii) 49 (iii) 64 (iv) 100

(v) 169 (vi) 81 (vii) 256 (viii) 144

Finding Square Root by Prime Factorisation

To find the square root of a perfect square by prime factorisation, we go through the following steps:

I. Write down the prime factors of the given number.

II. Make pairs of prime factors such that both the factors in each pair are equal.

III. Write one factor from each pair.

IV. Find the product of the above factors.

V. The product is the required square root.

Let us now take some examples to find the square root by prime factorisation.

Example 3: Find the square root of 1156.

Solution:

$$1156 = \underline{2 \times 2} \times \underline{17 \times 17}$$

$$\sqrt{1156} = \sqrt{\underline{2 \times 2} \times \underline{17 \times 17}}$$

$$\sqrt{1156} = 2 \times 17$$

$$= 34$$

2	1156
2	578
17	289
17	17
	1

Therefore, the square root of 1156 is 34.

Example 4: Find the square root of 11025.

3	11025
3	3675
5	1225
5	245
7	49
7	7
	1

Solution:

$$11025 = \underline{3 \times 3} \times \underline{5 \times 5} \times \underline{7 \times 7}$$

$$\sqrt{11025} = \sqrt{\underline{3 \times 3} \times \underline{5 \times 5} \times \underline{7 \times 7}}$$

$$\sqrt{11025} = 3 \times 5 \times 7$$

$$= 105$$

Therefore, the square root of 11025 is 105.

Example 5: Find the smallest number by which 9408 must be divided so that it becomes a perfect square. Also, find the square root of the number so obtained.

2	9408
2	4704
2	2352
2	1176
2	588
2	294
7	147
7	21
3	3
	1

Solution:

$$9408 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7} \times 3$$

We observe that prime factor 3 does not form a pair.

Therefore, we must divide the number by 3 so that the quotient becomes a perfect square.

$$\therefore \quad \frac{9408}{3} = 3136$$

$$3136 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (7 \times 7)$$

Now, each prime factor occurs in pairs. Therefore, the required smallest number is 3.

$$\therefore \quad \sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$$

Worksheet 3

1. Find the square root of each of the following by prime factorisation.

(i) 225 (ii) 441 (iii) 529 (iv) 40000

(v) 7744 (vi) 8281 (vii) 4096 (viii) 28900

2. Find the smallest number by which 1100 must be multiplied so that the product becomes a perfect square. Also, find the square root of the perfect square so obtained.

3. By what smallest number must 180 be multiplied so that it becomes a perfect square? Also, find the square root of the number so obtained.

4. **Find the smallest number by which 3645 must be divided so that it becomes a perfect square. Also, find the square root of the resulting number.**

5. **A gardener planted 1,521 trees in rows such that the number of rows was equal to the number of plants in each row. Find the number of rows.**

6. **An officer wants to arrange 2,02,500 cadets in the form of a square. How many cadets were there in each row?**

7. **The area of a square field is 5184 m^2. A rectangular field, whose length is twice its breadth, has its perimeter equal to the perimeter of the square field. Find the area of the rectangular field.**

8. **Find the value of $\sqrt{47089} + \sqrt{24336}$**

Finding Square Root by Long Division Method

When numbers are very large, the method of finding their square roots by prime factorisation becomes lengthy. So, we use long division method. Consider the following steps to find the square root of any number, say 1521.

Step 1: Mark off the digits in pairs starting with the ones digit. Each pair and remaining one digit (if it is there) is called a **period**.

$$\overline{)\,\overline{15}\,\overline{21}}$$

Step 2: Think of the largest number whose square is either equal to or just less than the first period starting from the left. This digit is the quotient as well as the divisor. Put the quotient above the period and write the product of divisor and quotient just below the first period.

$$\begin{array}{r|l} & 3 \\ \hline 3 & \overline{15}\,\overline{21} \\ & 9 \\ \hline \end{array}$$

$$\begin{array}{r|l} & 3 \\ \hline 3 & \overline{15}\,\overline{21} \\ & 9 \\ \hline & 6 \end{array}$$

Step 3: Find the remainder (6 in this case).

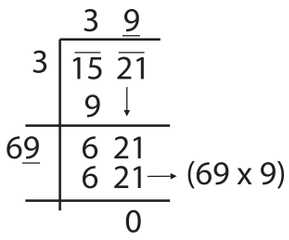
Step 4: Bring down the next pair of digits (i.e. second period) to the right of the remainder. This becomes the new dividend (i.e. 621).

$$\begin{array}{r|l} & 3 \\ \hline 3 & \overline{15}\,\overline{21} \\ & 9\downarrow \\ \hline & 6\,21 \end{array}$$

Step 5: Double the current quotient and enter it as divisor with a blank on its right.

$$\begin{array}{r|l} & 3 \\ \hline 3 & \overline{15}\,\overline{21} \\ & 9 \\ \hline (3 \times 2) \leftarrow 6\,\_ & 6\,21 \end{array}$$

Step 6: Guess a largest possible digit to fill the blank which also becomes the new digit in the quotient such that when the new digit is multiplied to the new divisor, the product is either less than or equal to the dividend.



Step 7: Now, subtract the product of new divisor and the new digit from the new dividend.

Step 8: If the remainder is zero and no period is left, then we stop and the current quotient is the square root of the given number (like in this case). So here, $\sqrt{1521} = 39$.

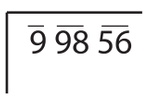
And if the remainder is non-zero, then repeat the Steps from 5 to 8 till all the periods have been taken care of.

Let us look at another example.

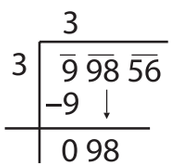
Example 6: Find $\sqrt{99856}$

Solution:

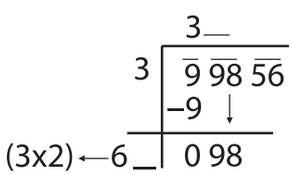
Step 1:



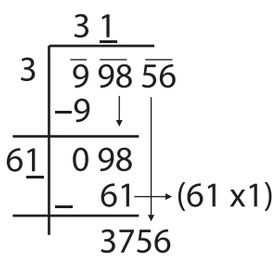
Step 2:



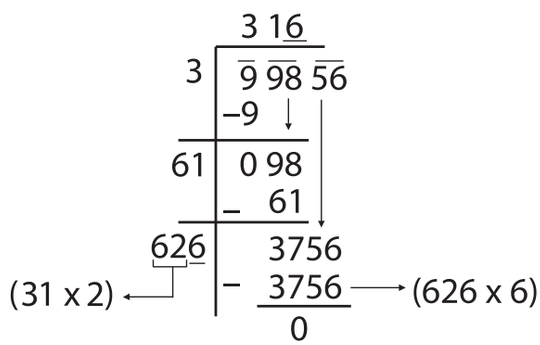
Step 3:



Step 4:



Step 5:

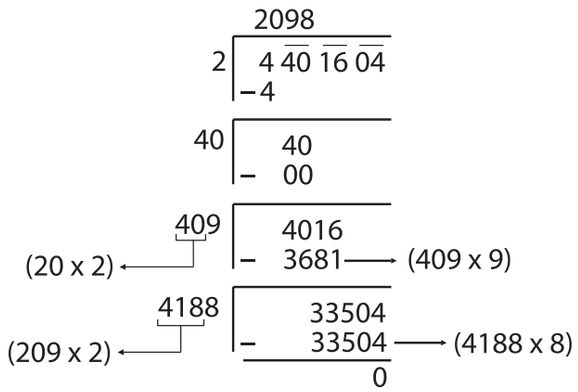


$\therefore \quad \sqrt{99856} = 316$

Now, let us try to understand long division method of square roots by some more examples.

Example 7: Find the square root of 4401604.

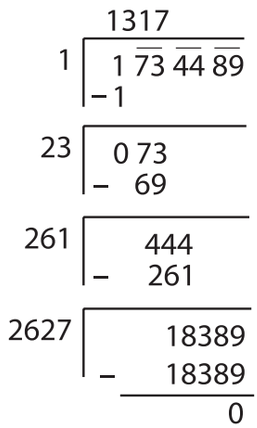
Solution:



$\therefore \quad \sqrt{4401604} = 2098$

Example 8: Find the square root of 1734489 by long division method.

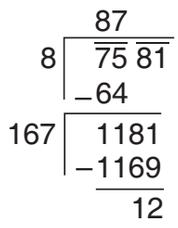
Solution: Apply long division method to find square root of 1734489.



$\therefore \quad \sqrt{1734489} = 1317$

Example 9: Find the least number which must be subtracted from 7581 to obtain a perfect square. Find the perfect square and its square root.

Solution:



$\therefore$ 12 should be subtracted from 7581 to make it a perfect square.

Hence, the perfect square = 7581 – 12 = 7569

and $\sqrt{7569} = 87$

Example 10: What least number must be added to 5607 to make the sum a perfect square? Find the perfect square and its square root.

Solution: Try to find the square root of 5607.

$$\begin{array}{r|l} & 74 \\ 7 & \overline{56}\ \overline{07} \\ & -49 \\ \hline 144 & 707 \\ & -576 \\ \hline & 131 \end{array} \qquad \begin{array}{r|l} & 75 \\ 7 & \overline{56}\ \overline{07} \\ & -49 \\ \hline 145 & 707 \\ & -725 \\ \hline & -18 \end{array}$$

We observe that $(74)^2 < 5607 < (75)^2$

$\therefore$ 5607 is (725 – 707) = 18 less than $(75)^2$.

So, we must add 18 to 5607 to make it a perfect square.

Hence, the perfect square = 5607 + 18 = 5625 and $\sqrt{5625} = 75$

Worksheet 4

1. Find the square root of the following numbers by the long division method.

(i)	9801	(ii)	6561	(iii)	390625	(iv)	108241
(v)	363609	(vi)	120409	(vii)	1471369	(viii)	57121

2. Find the least number which must be subtracted from 6203 to obtain a perfect square. Also, find square root of the number so obtained.

3. Find the greatest number of six digits which is a perfect square. Find the square root of this number.

4. Find the least number which must be added to 6203 to obtain a perfect square. Also, find the square root of the number so obtained.

5. Find the least number of six digits which is a perfect square. Find the square root of this number.

6. Find the value of $\sqrt{64432729} - \sqrt{9653449}$

SQUARE ROOT OF A RATIONAL NUMBER

In this Chapter, we shall be taking some examples to understand the rules of finding the square root of rational numbers.

Example 11: Find $\sqrt{49\times 25}$ and show that it is equal to $\sqrt{49}\times\sqrt{25}$.

Solution: $\sqrt{49\times 25} = \sqrt{7^2\times 5^2}$

$= \sqrt{7^2\times 5^2}$ [We know $a^m \times b^m = (ab)^m$]

$= \sqrt{(35)^2} = 35 = 7\times 5$

$= \sqrt{49}\times\sqrt{25}$

Rule I: For perfect squares *a* and *b*, $\sqrt{a\times b} = \sqrt{a}\times\sqrt{b}$.

Example 12: Consider $\sqrt{\frac{49}{25}}$ and $\frac{\sqrt{49}}{\sqrt{25}}$ and find out whether they are equal.

Solution: $\sqrt{\frac{49}{25}} = \sqrt{\frac{7^2}{5^2}}$

$\left\{\text{We know } \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m},\ b\neq 0\right\} = \sqrt{\left(\frac{7}{5}\right)^2} = \frac{7}{5}$

Thus, $\sqrt{\frac{49}{25}} = \frac{7}{5} = \frac{\sqrt{49}}{\sqrt{25}}$

Also, $\frac{\sqrt{49}}{\sqrt{25}} = \frac{\sqrt{7^2}}{\sqrt{5^2}}$

$= \frac{7}{5}$

Rule II: For perfect squares *a* and *b*, where $b \neq 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Now, let us apply Rule I and II and solve some examples.

Example 13: Find the value of $\frac{\sqrt{243}}{\sqrt{867}}$

Solution: $\frac{\sqrt{243}}{\sqrt{867}} = \sqrt{\frac{243}{867}}$ (using Rule II)

$= \sqrt{\frac{81}{289}}$ (cancel the common factor 3)

$$= \frac{\sqrt{81}}{\sqrt{289}}$$

$$= \frac{\sqrt{9^2}}{\sqrt{17^2}}$$

$$= \frac{9}{17}$$

Thus, the value of $\frac{\sqrt{243}}{\sqrt{867}} = \frac{9}{17}$.

Example 14: Find the square root of:

(i) $1\frac{56}{169}$ (ii) 14400

Solution: (i) $\sqrt{1\frac{56}{169}} = \sqrt{\frac{225}{169}}$

$= \frac{\sqrt{225}}{\sqrt{169}} = \frac{\sqrt{15^2}}{\sqrt{13^2}}$ (by Rule II)

$= \frac{15}{13}$

$= 1\frac{2}{13}$

(ii) $\sqrt{14400} = \sqrt{144 \times 100}$

$= \sqrt{144} \times \sqrt{100}$ (by Rule I)

$= \sqrt{12^2} \times \sqrt{10^2}$

$= 12 \times 10$

$= 120$

SQUARES OF TERMINATING DECIMALS

Observe some squares.

4^2	=	16	and	$(0.4)^2$	=	0.16
12^2	=	144	and	$(1.2)^2$	=	1.44
15^2	=	225	and	$(0.15)^2$	=	0.0225
18^2	=	324	and	$(0.18)^2$	=	0.0324

From the above examples, we observe that the square of a decimal consists of twice as many decimal places as given in the number.

Example 15: Find the square of:

(i) 0.41 (ii) 2.6 (iii) 0.25 (iv) 0.009

Solution: (i) $(0.41)^2 = 0.1681$

(ii) $(2.6)^2 = 6.76$

(iii) $(0.25)^2 = 0.0625$

(iv) $(0.009)^2 = 0.000081$

Square Roots of Numbers in Decimal Form (which are perfect squares)

Let us find the square root of a decimal number.

Example 16: Find the square root of 147.1369

Solution:

$$
\begin{array}{r|l}
 & 12.13 \\
1 & \overline{1}\,\overline{47}\,.\,\overline{13}\,\overline{69} \\
 & \underline{-1} \\
22 & 0\,47 \\
 & \underline{-\,44} \longrightarrow (22 \times 2) \\
241 & 313 \\
 & \underline{-\,241} \longrightarrow (241 \times 1) \\
2423 & 7269 \\
 & \underline{-\,7269} \longrightarrow (2423 \times 3) \\
 & 0
\end{array}
$$

Therefore, $\sqrt{147.1369} = 12.13$

From the above example, the steps of finding out square roots of numbers in decimal form are clear.

Step 1: In the whole number part, make pairs from the right. But in the decimal part, make pairs from the left.

Step 2: Then, find square root as in the case of long division method.

Step 3: Place the decimal point as soon as the integral part comes to an end.

Observe that above steps are taken in the following example also.

Example 17: Find the square root of 0.00059049

Solution:

$$
\begin{array}{r|l}
 & 0.0243 \\
\hline
0 & 0.\overline{00}\ \overline{05}\ \overline{90}\ \overline{49} \\
 & -\,00 \\
\hline
2 & \quad 05 \\
 & -\ \ 4 \\
\hline
44 & \quad\ 190 \\
 & -176 \\
\hline
483 & \quad\ \ 1449 \\
 & -1449 \\
\hline
 & \quad\ \ 0
\end{array}
$$

Therefore, $\sqrt{0.00059049} = 0.0243$

Approximate Value of the Square Roots of Natural Numbers (which are not perfect squares)

Long division method is also used to find approximate square roots of numbers or decimals up to certain decimal places. Let us look at the following examples.

Example 18: Find the square root of 3 up to three decimal places.

Solution: To find the number up to three decimal places which is equal to $\sqrt{3}$, we add three pairs of zeroes (six zeroes) to the right of decimal point.

$$
\begin{array}{r|l}
 & 1.732 \\
\hline
1 & 3.\overline{00}\ \overline{00}\ \overline{00} \\
 & -1 \\
\hline
27 & \quad 200 \\
 & -189 \\
\hline
343 & \quad\ 1100 \\
 & -1029 \\
\hline
3462 & \quad\ \ 7100 \\
 & -6924 \\
\hline
 & \quad\ \ 176
\end{array}
$$

Hence, $\sqrt{3} = 1.732$ up to three decimal places.

Example 19: Find the square root of $2\frac{1}{5}$ correct to two places of decimal.

Solution: $2\frac{1}{5} = \frac{11}{5} = 2.2$

$\sqrt{2\frac{1}{5}} = \sqrt{2.2}$

$$\begin{array}{r|l} & 1.483 \\ \hline 1 & 2.\overline{20}\ \overline{00}\ \overline{00} \\ & \underline{-1} \\ \hline 24 & 120 \\ & \underline{-96} \\ \hline 288 & 2400 \\ & \underline{-2304} \\ \hline 2963 & 9600 \\ & \underline{-8889} \\ & 711 \end{array}$$

$\therefore \sqrt{2\frac{1}{5}} = \sqrt{2.2} = 1.483$

$\cong$ 1.48 (correct to two places of decimal)

Note: We were required to find the square root of $2\frac{1}{5}$ correct to two places of decimal. Here, we have found the square root up to three places of decimal. In the third place, we have 3 (<5) and therefore, in the final result, 3 is ignored.

Square Root of Other Numbers (not perfect squares) by Estimation

It is easy to work out the square root of a perfect square, but it is really hard to work out the square root of other numbers. Well, in such cases, we need to **estimate** the square root. Let us do some examples.

Example 20: Find the square root of 10 by estimation.

Solution: The perfect squares near to 10 are 9 and 16,

i.e. $9 < 10 < 16$ or

$3^2 < 10 < 4^2$

So, we can guess that the answer is between 3 and 4, i.e. $3 < \sqrt{10} < 4$

Let us try with 3.5 as $3 < 3.5 < 4$

But $3.5 \times 3.5 = 12.25 > 10$

i.e. $3^2 < 10 < (3.5)^2$

Let us further reduce the number 3.5 to 3.2

So, $3.2 \times 3.2 = 10.24 > 10$

i.e. $3^2 < 10 < (3.2)^2$

Let us try with 3.1 so that

$3.1 \times 3.1 = 9.61$

So, we can say $9.61 < 10 < 10.24$

or $(3.1)^2 < 10 < (3.2)^2$

But 10.24 is much closer to 10 as compared to 9.61.

So, we can say $\sqrt{10}$ is 3.2 approximately.

Example 21: Find the square root of 410 by estimation.

Solution: The perfect square near to 410 are 400 and 441

i.e. $400 < 410 < 441$

$20^2 < 410 < 21^2$

We guess the answer is between 20 and 21

Let us try with 20.3 as $20 < 20.3 < 21$

But $(20.3)^2 = 412.09 > 410$

i.e. $20^2 < 410 < (20.3)^2$

Let us try 20.2

$20.2 \times 20.2 = 408.04$

$408.04 < 410 < 412.09$

$(20.2)^2 < 410 < (20.3)^2$

We take $\sqrt{410}$ as 20.2 approximately.

Worksheet 5

1. Find the square root of the following fractions.

(i) $\frac{324}{361}$ (ii) $\frac{441}{961}$ (iii) $5\frac{19}{25}$ (iv) $21\frac{51}{169}$

(v) $\frac{5625}{441}$ (vi) $7\frac{18}{49}$ (vii) $23\frac{394}{729}$ (viii) $35\frac{85}{1444}$

2. Find the value of:

(i) $\sqrt{0.0009}$ (ii) $\sqrt{0.0081}$ (iii) $\sqrt{0.012321}$ (iv) $\sqrt{7.29}$

3. Find the square root of:

(i) 0.053361 (ii) 0.00053361 (iii) 150.0625

(iv) 0.374544 (v) 610.09

4. Find the square root of the following (correct to three decimal places).

(i) 7 (ii) 2.5 (iii) $2\frac{1}{12}$ (iv) $367\frac{2}{7}$

5. Estimate the value of the following to the nearest to one decimal place.

(i) $\sqrt{90}$ (ii) $\sqrt{150}$ (iii) $\sqrt{600}$ (iv) $\sqrt{1000}$

6. Devika has a square piece of cloth of area 9 m^2 and she wants to make 16 square-shaped scarves of equal size out of it. What should possibly be the length of the side of the scarf that can be made out of this piece?

7. The area of a square plot is 800 m^2. Find the estimated length of the side of the plot.

Value Based Questions

1. Priya wants to wish her teacher on Teacher's Day by giving her a self-made greeting card. She chooses a pink coloured square sheet of paper. A side of that paper measures 19.5 cm.

(a) Find the area of paper she chooses for the card.

(b) What act of Priya did you like?

2. The students of Class-VIII B of a school donated ₹ 2,304 for the Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class.

(a) Find the number of students in VIII B.

(b) What quality of the students do you appreciate here?

Brain Teasers

1.A. Tick (✓) the correct option.

(a) The difference between the squares of two consecutive number is equal to their—

(i) difference (ii) sum (iii) product (iv) quotient

(b) What will be the digit in the thousands place of $(1111)^2$?

(i) 3 (ii) 4 (iii) 2 (iv) 1

(c) Perfect squares cannot have 2, 3, __ and __ in its ones place.

(i) 1, 7 (ii) 5, 6 (iii) 7, 8 (iv) 7, 9

(d) The smallest number by which 72 must be divided to make it a perfect square is—

(i) 4 (ii) 5 (iii) 3 (iv) 2

(e) The square root of 3.052009 has _____ decimal places.

(i) 3 (ii) 4 (iii) 5 (iv) 1

B. Answer the following questions.

(a) How many non–square numbers are there between 13^2 and 14^2?

(b) Write the first four triangular numbers.

(c) Is 5, 7, 9 a Pythagorean triplets? Why? Justify.

(d) Find $\sqrt{9}$ by repeated subtraction method.

(e) Find the measure of the side of a square handkerchief of area 324 cm^2.

2. **Find the square root of 10 correct to four places of decimal.**

3. **Find the values of : $\sqrt{3.1428}$ and $\sqrt{0.31428}$ correct to three decimal places.**

4. **Simplify:**

   (i) $\dfrac{\sqrt{0.0441}}{\sqrt{0.000441}}$ (ii) $\sqrt{49} + \sqrt{0.49} + \sqrt{0.0049}$

5. **The area of a square field is $101\dfrac{1}{400}$ m^2. Find the length of one side of the field.**

6. **What is that number which when multiplied by itself gives 227.798649?**

7. **In a lecture hall, 8,649 students are sitting in such a manner that there are as many students in a row as there are rows in the lecture hall. How many students are there in each row of the lecture hall?**

8. **A General wishing to draw up his 64,019 men in the form of a square found that he had 10 men extra. Find the number of men in the front row.**

HOTS

1. **The cost of levelling a square lawn at ₹ 15 per square metre is ₹ 19,935. Find the cost of fencing the lawn at ₹ 22 per metre.**

2. **If $\sqrt{2}$ = 1.414, $\sqrt{5}$ = 2.236 and $\sqrt{3}$ = 1.732, find the value of:**

   (i) $\sqrt{72} + \sqrt{48}$ (ii) $\sqrt{\dfrac{125}{64}}$

Enrichment Questions

1. **The product of two numbers is 1296. If one number is 16 times the other, find the number.**

2. **Find the value of $\sqrt{50625}$ and hence the value of $\sqrt{506.25} + \sqrt{5.0625}$.**

3. **Write a Pythagorean triplet if one number is 14.**

You Must Know

1. The square of a number is that number raised to the power 2.

2. A square number is never negative.

3. A number ending in 2, 3, 7 or 8 is never a perfect square.

4. (i) Squares of even numbers are even.

   (ii) Squares of odd numbers are odd.

5. A perfect square number leaves a remainder 0 or 1 on division by 3.

6. There are no natural numbers p and q such that $p^2 = 2q^2$.

7. If a and b are perfect squares ($b \neq 0$), then

   $$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

   and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

8. The square root of a perfect square can be obtained by —

   (i) finding prime factors.

   (ii) long division method.

9. The pairing of numbers in the division method starts from a decimal point. For the integral part, it goes from right to left and for the decimal part, it goes from left to right.

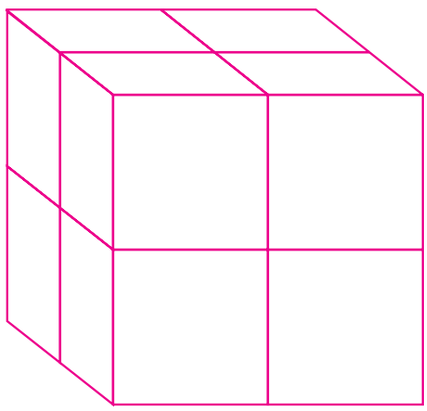
10. If p and q are not perfect squares, then to find $\sqrt{\frac{p}{q}}$, we express $\frac{p}{q}$ as a decimal and then apply division method to find the square root.

CHAPTER 2

CUBES AND CUBE ROOTS

INTRODUCTION

In this Chapter, we shall confine ourselves to exponent **three**, that is $n^3 = n \times n \times n$. It is called **cube** of n.



Now, let us take a cube of 2 units and examine it. Each side of a cube is of 2 units.

Therefore, the volume of a cube having 2 units

$= 2 \times 2 \times 2 = 2^3$ cubic units

$= 8$ cubic units

We say that 2 to the power of 3 or 2 cubed is 8.

Now, let us study cubes of rational numbers and cube roots of those rational numbers which are perfect cubes. In the previous chapter, we have taken only the square roots of positive rational numbers, whereas in this Chapter, we shall be studying the cube roots of positive as well as negative rational numbers.

CUBES

Let us observe the following products:

$2 \times 2 \times 2 = 8 \longrightarrow 8 = 2^3$

$3 \times 3 \times 3 = 27 \longrightarrow 27 = 3^3$

Similary, $a \times a \times a = a^3$

So, we conclude that—

The cube of a number is product of a number multiplied by itself three times and is read as the number raised to the power 3.

Observe the cubes of following numbers.

$$4^3 = 4 \times 4 \times 4 = 64$$

$$8^3 = 8 \times 8 \times 8 = 512$$

$$12^3 = 12 \times 12 \times 12 = 1728$$

$$17^3 = 17 \times 17 \times 17 = 4913$$

We see that 64, 512, 1728 and 4913 are cubes of some positive integers. Such numbers are called **perfect cubes**.

An integer n is a perfect cube if there is an integer m such that $n = m \times m \times m$.

In order to check whether a given number is a perfect cube, we follow the given steps:

Step 1: Express the given number as a product of prime factors.

Step 2: Group the factors in triplets such that all three factors in each triplet are the same.

Step 3: If some prime factors are left ungrouped, the given number is not a perfect cube.

Now, let us examine some numbers for being perfect cubes.

Example 1: Is 216 a perfect cube?

Solution: Resolve 216 into prime factors.

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

Group them in like factors.

$$216 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$$

$$= 2^3 \times 3^3$$

2	216
2	108
2	54
3	27
3	9
3	3
	1

Prime factors of 216 can be grouped into triplets and no factor is left over.

$\therefore$ 216 is a perfect cube.

Example 2: Is 1323 a perfect cube?

Solution: Resolve 1323 into prime factors.

$$1323 = \underline{3 \times 3 \times 3} \times 7 \times 7$$

While grouping the factors, we are left with 7×7.

Therefore, 1323 is not a perfect cube.

3	1323
3	441
3	147
7	49
7	7
	1

Example 3: What is the smallest number by which 3087 may be multiplied so that the product is a perfect cube?

3	3087
3	1029
7	343
7	49
7	7
	1

Solution: Find prime factors of 3087.

$$3087 = 3 \times 3 \times \underline{7 \times 7 \times 7}$$

On grouping the factors, we find that 3×3 is left out.

So, if we multiply 3087 by 3, the factors would be
$\underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$. [3 appears three times]

Hence, 3087 must be multiplied by 3 to make it a perfect cube.

Example 4: What is the smallest number by which 3087 must be divided so that the quotient is a perfect cube?

3	3087
3	1029
7	343
7	49
7	7
	1

Solution: Resolve 3087 into prime factors.

$$3087 = 3 \times 3 \times \underline{7 \times 7 \times 7}$$

On grouping the factors, we find that 3×3 is left out.

So, we divide 3087 by $3 \times 3 = 9$.

The quotient then would be $7 \times 7 \times 7$, which is a perfect cube.

$\therefore$ 3087 must be divided by 9 so that the quotient becomes a perfect cube.

Properties of Cubes of Numbers

Let us now keep in mind, cubes of some numbers, i.e.

$2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, $6^3 = 216$, ...

We will observe that the following properties are true:

I. The cube of an even number is always even, whereas the cube of an odd number is always odd.

II. The cube of any multiple of 2 is divisible by 8.

$(4)^3 = 64$ which is divisible by 8.

$(12)^3 = 1728$ which is divisible by 8.

III. The cube of any multiple of 3 is divisible by 27.

$(9)^3 = 729$ which is divisible by 27.

$(12)^3 = 1728$ which is divisible by 27.

IV. The cube of a negative number is always negative.

$(-2)^3 = (-2) \times (-2) \times (-2) = 4 \times (-2) = -8$

$(-5)^3 = (-5) \times (-5) \times (-5) = 25 \times (-5) = -125$

V. The cube of a positive number is always positive.

VI. The cube of a rational number is equal to the cube of its numerator divided by the cube of its denominator.

$$\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{27}{125}$$

Worksheet 1

1. Find the cubes of:

(i) 8 (ii) 13 (iii) 17 (iv) 1.3

(v) 0.06 (vi) 0.4 (vii) $\frac{2}{3}$ (viii) – 7

(ix) – 9 (x) – 12

2. Which of the following numbers are perfect cubes?

(i) 4096 (ii) 108 (iii) 392

(iv) – 27000 (v) $\frac{-64}{1331}$

3. Find the smallest number by which 2560 must be multiplied so that the product is a perfect cube.

4. Find the smallest number by which 8788 be divided so that the quotient is a perfect cube.

5. Write True or False for the following statements.

(i) 650 is not a perfect cube. _____

(ii) Perfect cubes may end with two zeroes. _____

(iii) Perfect cubes of odd numbers may not always be odd. _____

(iv) Cube of negative numbers are negative. _____

(v) For a number to be a perfect cube, it must have prime factors in triplets. _____

CUBE ROOTS

We know that

$$2^3 = 2 \times 2 \times 2 = 8$$

We say cube root of 8 is 2

We write $\sqrt[3]{8} = 2$

Let us see more examples.

$$3^3 = 27 \longrightarrow \sqrt[3]{27} = 3$$
$$4^3 = 64 \longrightarrow \sqrt[3]{64} = 4$$
$$5^3 = 125 \longrightarrow \sqrt[3]{125} = 5$$
$$(-10)^3 = -1000 \longrightarrow \sqrt[3]{-1000} = -10$$
$$(-7)^3 = -343 \longrightarrow \sqrt[3]{-343} = -7$$

In general if, $m^3 = n$ then $\sqrt[3]{n} = m$.

Hence, cube root of a given number n is that number m whose cube is n.

Note: '$\sqrt[3]{\ }$' symbol denotes cube root whereas '$\sqrt{\ }$' denotes square root.

Some Patterns Involving Cubic Numbers

I. Let us look at some interesting patterns expressing cubic numbers.

$$1^3 = 1$$
$$2^3 = 3 + 5$$
$$3^3 = 7 + 9 + 11$$
$$4^3 = 13 + 15 + 17 + 19$$
$$5^3 = 21 + 23 + 25 + 27 + 29$$

and so on.

What do we observe from the above pattern?

We see that the cube of a number n can be expressed as the sum of the n odd consecutive numbers.

II. Let us see another pattern.

$2^3 - 1^3 = 1 + 2 \times 1 \times 3$

$3^3 - 2^3 = 1 + 3 \times 2 \times 3$

$4^3 - 3^3 = 1 + 4 \times 3 \times 3$

$5^3 - 4^3 = 1 + 5 \times 4 \times 3$

$6^3 - 5^3 = 1 + 6 \times 5 \times 3$

and so on.

III. Now, let us look at this pattern.

$1^3 = 1$

$2^3 - 1^3 = 8 - 1 = 7$

$3^3 - 2^3 = 27 - 8 = 19$

$4^3 - 3^3 = 64 - 27 = 37$

$5^3 - 4^3 = 125 - 64 = 61$

$6^3 - 5^3 = 216 - 125 = 91$

$\therefore$

$1^3 = 1$

$2^3 = 1 + 7$

$3^3 = 1 + 7 + 19$

$4^3 = 1 + 7 + 19 + 37$

$5^3 = 1 + 7 + 19 + 37 + 61$

$6^3 = 1 + 7 + 19 + 37 + 61 + 91$

and so on.

The last number in each case, that is 1, 7, 19, 37, 61, ... may be obtained by putting $n = 0, 1, 2, 3, 4, \ldots\ldots$ in $[1 + n\,(n + 1) \times 3]$.

Cube Root by Prime Factorisation

Let us find the cube root of 74088.

Resolve 74088 into prime factors.

$74088 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$

$\therefore \quad \sqrt[3]{74088} = 2 \times 3 \times 7 = 42$

The steps involved to find the cube root by prime factorisation are as under:

Step 1: Resolve the given perfect cube into its prime factors.

Step 2: Group the factors into triplets.

Step 3: Take one factor out of each triplet.

Step 4: Multiply all the factors taken from the triplets. Their product will be the required cube root.

Let us follow the given rule to find cube root of some numbers through prime factorisation.

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

Example 5: Find the cube root of – 2744.

Solution: Resolve 2744 into prime factors.

2	2744
2	1372
2	686
2	343
7	49
7	7
	1

$$2744 = \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$$

$$\therefore \quad \sqrt[3]{2744} = 2 \times 7 = 14$$

Hence, $\sqrt[3]{-2744} = -\sqrt[3]{2744} = -14$

Thus, from the above example we observe that—

For a positive integer x,

$$-x = -1 \times x$$

$$\therefore \quad \sqrt[3]{-x} = \sqrt[3]{(-1) \times x} = \sqrt[3]{-1} \times \sqrt[3]{x} \qquad \because \sqrt[3]{a \times b} = \sqrt[3]{a} \times \sqrt[3]{b}$$

However, $\sqrt[3]{-1} = -1$ as $(-1)^3 = -1 \times -1 \times -1 = -1$

$$\therefore \quad \sqrt[3]{-x} = -\sqrt[3]{x}$$

Example 6: Find the cube root of – 3375.

Solution: We have, $\sqrt[3]{-3375} = -\sqrt[3]{3375}$

5	3375
5	575
5	135
3	27
3	9
3	3
	1

Resolve 3375 into prime factors.

$$3375 = \underline{5 \times 5 \times 5} \times \underline{3 \times 3 \times 3}$$

$$\sqrt[3]{3375} = 5 \times 3 = 15$$

$$\therefore \quad \sqrt[3]{-3375} = -15$$

Cube Root of Rational Numbers

Now, let us find $\sqrt[3]{\frac{8}{125}}$

$$\sqrt[3]{\frac{8}{125}} = \sqrt[3]{\frac{2\times2\times2}{5\times5\times5}} = \sqrt[3]{\left(\frac{2}{5}\right)^3}$$

or $$\sqrt[3]{\frac{8}{125}} = \sqrt[3]{\left(\frac{2}{5}\right)^3} = \frac{2}{5}$$

$$\frac{\sqrt[3]{8}}{\sqrt[3]{125}} = \frac{\sqrt[3]{2^3}}{\sqrt[3]{5^3}}$$

$$= \frac{2}{5}$$

Thus, if x and y (where $y \neq 0$) are perfect cubes,

Then, $$\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}}$$

Example 7: Find the cube root of $\frac{729}{1000}$.

Solution: $\sqrt[3]{\frac{729}{1000}} = \frac{\sqrt[3]{729}}{\sqrt[3]{1000}}$

Resolving 729 and 1000 into prime factors.

3	729
3	243
3	81
3	27
3	9
3	3
	1

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$729 = \underline{3\times3\times3} \times \underline{3\times3\times3}$$

$$1000 = \underline{2\times2\times2} \times \underline{5\times5\times5}$$

$\therefore$ $\sqrt[3]{729} = 3\times3 = 9$

and $\sqrt[3]{1000} = 2\times5 = 10$

Hence, $$\sqrt[3]{\frac{729}{1000}} = \frac{\sqrt[3]{729}}{\sqrt[3]{1000}} = \frac{9}{10}$$

Cube Root of a Number through Estimation

Let us find the cube root of a given number through estimation.

Consider the number 13824.

Step 1: Start making the groups of three digits starting from the right most digit of the number,

i.e. Second group First group

13 824

Step 2: From the first group 824, take the digit from ones place. This is 4, which will be the ones digit in the cube root of the given number (as $4^3 = \underline{64}$ so 4 in the ones place of the required cube root).

Step 3: Now, take other group, i.e. 13.

We know that $8 < 13 < 27$,

i.e. $2^3 < 13 < 3^3$

So we take the ones place of the smaller number, i.e. 2 as the tens place of the required cube root.

So $\sqrt[3]{13824} = 24$

Example 8: Find the cube root of 175616 through estimation.

Solution:

Step 1: Form the groups of three digits starting from the right most digit of 175616.

II	I
175	616

Step 2: Let us consider the first group, i.e. 616. It has 6 in its ones place. Now, $6^3 = 216$ so 216 also has 6 in its place. So this gives the number at ones place of the required cube root.

Step 3: Now, consider the second group, i.e. 175

Now, $125 < 175 < 216$,

i.e. $5^3 < 175 < 6^3$

So the smaller number between 5 and 6 is 5 which qualifies for the tens place of the cube root.

$\therefore \quad \sqrt[3]{175616} = 56$

Worksheet 2

1. Find the cube root of the following by prime factorisation.

(i) 5832 (ii) 1728 (iii) 216000 (iv) 21952

2. Find the cube root of the following integers.

(i) – 1728 (ii) – 2744000 (iii) – 474552 (iv) – 5832

3. Evaluate:

(i) $\sqrt[3]{8\times125}$ (ii) $\sqrt[3]{3375\times(-729)}$ (iii) $\sqrt[3]{4^3\times5^3}$

4. Find the cube root of the following rational numbers.

(i) $\frac{4913}{3375}$ (ii) $\frac{-512}{343}$ (iii) $\frac{-686}{-2662}$

5. By which smallest number must 5400 be multiplied to make it a perfect cube?

6. Find the smallest number by which 16384 be divided so that the quotient may be a perfect cube.

7. Find the cube root of the following numbers through estimation.

(i) 10648 (ii) 15625 (iii) 110592 (iv) 91125

Value Based Questions

1. Students of a school collected provisions like rice, pulses, etc., for the flood affected people of Madhya Pradesh. These provisions were packed in six cubical cartons each of side measuring 65 cm.

(a) Find the volume of cartons packed.

(b) What values do you learn from the students?

2. A school decided to award prizes to students for three values—discipline, cleanliness of environment and regularity in attendance. The number of students getting prizes in the three categories are in the ratio 1 : 2 : 3. If product of ratios is 162, then—

(a) Find the number of students getting prizes for each value.

(b) Name any other two values that you can inculcate.

Brain Teasers

1.A. Tick (✓) the correct option.

(a) Cube of 0.1 is equal to—

(i) 1.11 (ii) 0.001 (iii) 0.101 (iv) 0.01

(b) The smallest number by which 1944 should be multiplied so that it becomes a perfect cube is—

(i) 3 (ii) 2 (iii) 5 (iv) 4

(c) Value of $\sqrt[3]{1000000}$ is—

(i) 10 (ii) 1000 (iii) 100 (iv) none of these

(d) $\sqrt[3]{0.027} - \sqrt[3]{0.008}$ is equal to—

(i) 1 (ii) 0.1 (iii) 0.11 (iv) 0.09

(e) Cube of $\left(\frac{-1}{3}\right)$ is equal to—

(i) $\frac{1}{27}$ (ii) $\frac{-1}{9}$ (iii) $\frac{-1}{27}$ (iv) $\frac{1}{9}$

B. Answer the following questions.

(a) Find the number whose cube is 1728.

(b) Find the value of $\sqrt[3]{216 \times (-125)}$.

(c) Find the cube root of 0.000001

(d) What is the smallest number by which 1715 should be divided so that the quotient is a perfect cube?

(e) Evaluate : $\sqrt[3]{\frac{0.512}{0.343}}$

2. Prove that if a number is tripled, then its cube is 27 times the cube of the given number.

3. Write cubes of all natural numbers between 1 to 10 and observe the pattern.

4. Find the cubes of:

(i) 0.6 (ii) – 3.1 (iii) – 0.01

5. Find the value of the following cube roots.

(i) $\sqrt[3]{0.008}$ (ii) $\sqrt[3]{\frac{-64}{1331}}$ (iii) $\sqrt[3]{27 \times 2744}$

6. **Find the smallest number which when multiplied with 3600 will make the product a perfect cube. Further, find the cube root of the product.**

7. **Evaluate:** $\sqrt[3]{\frac{0.027}{0.008}} \div \sqrt{\frac{0.09}{0.04}} - 1$

8. **Guess the cube root of the following numbers.**

   (i) 6859 (ii) 12167 (iii) 32768

HOTS

1. **Evaluate :** $\sqrt[3]{288\sqrt[3]{72\sqrt[3]{27}}}$

2. **Three numbers are in the ratio 2 : 3 : 4. The sum of their cubes is 33957. Find the numbers.**

Enrichment Questions

1. **Find the cube root of 4741632 by estimation.**

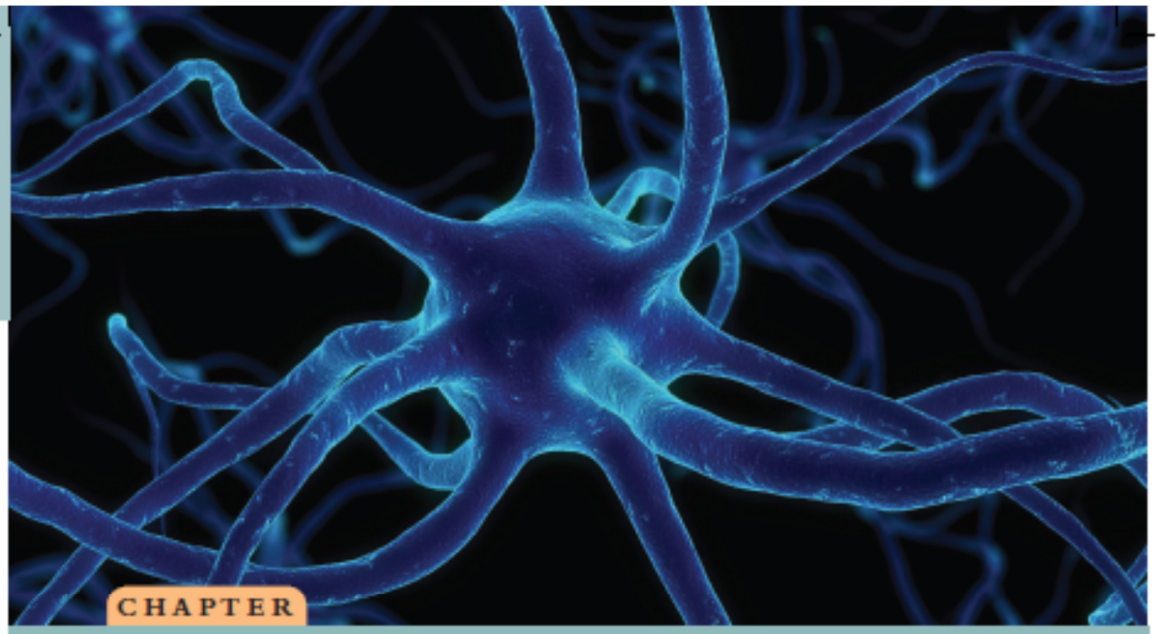
2. **Find the volume of a cube whose surface area is 150 m^2.**

You Must Know

1. The cube of a number is the number raised to the power 3.
2. The cube of an even natural number is even.
3. The cube of an odd natural number is odd.
4. The cube root of a number x is the number whose cube is x. It is denoted by $\sqrt[3]{x}$.
5. For any positive integer x, we have = $\sqrt[3]{-x} = -\sqrt[3]{x}$.
6. For any two integers a and b, we have,

   (i) $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$.

   (ii) $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}, b \neq 0.$



CHAPTER

1 The Cell – Its Structure and Functions

There are a large variety of organisms on this earth that are all distinct in their form and structure. However, they all possess similarity in their basic structure and functions. Just as a building is made up of bricks, similarly, the 'bodies' of all plants and animals are made up of cells. From microscopic bacteria, or *Amoeba*, to large organisms, like elephants, whales or gigantic trees, all are made up of 'cells', the basic units of all organisms.

Some cells exist as unicellular organisms (single-celled individuals) while others are a part of multicellular organisms. Certain basic functions, like nutrition, respiration, growth, development and reproduction, are performed by the cells in all organisms. These functions are essential for the survival of the organisms. We, therefore, regard the **cell** as the basic structural as well as functional unit of all living organisms.

In this Chapter, we will study about the variety in the shape, size, structure and functions of the cells of different organisms.

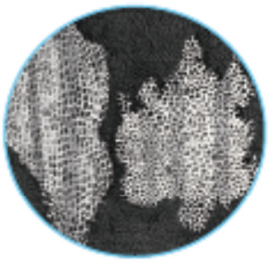
Discovery of the Cell

Cells are the basic 'structural unit' of all living beings. They remained undiscovered for a long time because the majority of the cells are too small to be seen by the unaided eye.

It was only after the advent of optical instruments, in the seventeenth century, that the cell was discovered and its basic features were studied.

Robert Hooke was the first scientist who, in 1665, observed thin slices of cork (obtained from the bark of a tree) through his self-designed microscope. He observed that they had honey-comb like structures consisting of little compartments (in Latin, *'cell'* means 'a little room'). It was later explained that these 'compartments' were actually 'dead cells', bound by a 'cell wall'.

Robert Hooke



'Honey-comb' structure of cork cells

The Cell

We now know that living organisms are made up of cells. The cells have the same basic structure, but they are different, with respect to their number, shape and size, in different living organisms.

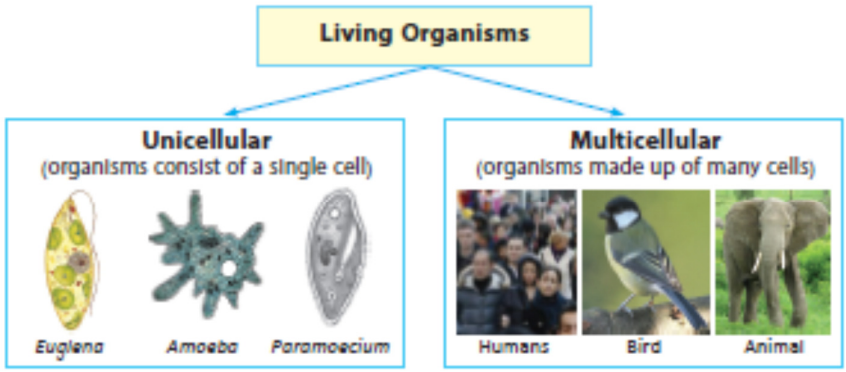
Do You Know?

The outermost layer of our skin consists of dead cells. This layer is shed periodically and is replaced by newer cells. You may be surprised to know that a person may lose about four kilograms of skin cells every year.

Variation in Cell Number, Shape and Size in Living Organisms

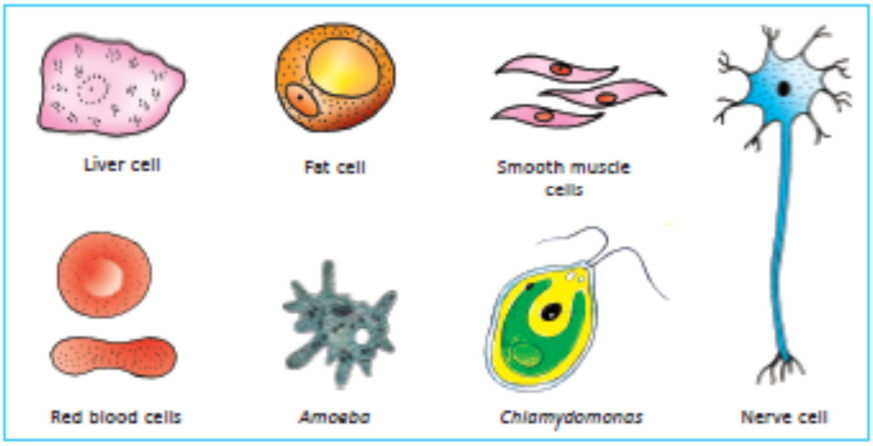
Cell Number

An *Amoeba* and an earthworm are of different sizes. This difference, in the size of the organism, is due to the number of cells present in them. While *Amoeba* is a living organism consisting of a single cell, an earthworm has millions of cells. Hence, on the basis of their 'number of cells', living organisms can be classified into two categories: **unicellular** and **multicellular**.



- ## Cell Shape

The shapes of cells differ not only in different organisms but also in different organs of the same organism. They may be oval, spherical, cuboidal, fibre-like or polygonal. These differences in shapes are due to their location and function in the tissue. For instance, a nerve cell has to transmit nerve impulses to organs located in different parts of the body. Hence, they possess a long fibre-like structure.



Variety in the shape of Cells

- ## Cell Size

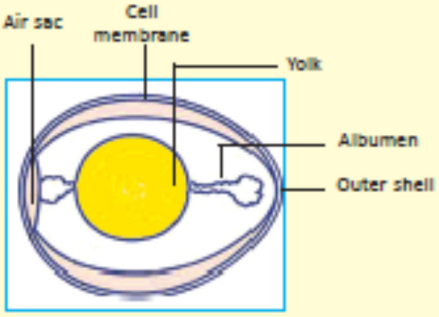
Cells vary considerably in their size. The smallest cell PPLO (Pleuro pneumonia-like organism), also called **mycoplasma**, is about 0.1 micron (denoted as 'μ') in diameter ($1\mu = 10^{-6}$m). The ostrich egg, considered to be the largest cell, is (nearly) 170 mm in diameter.

The hen's egg also represents a single cell; it is big enough to be seen with the unaided eye.

Activity 1

Take a hen's egg. Gently break its shell and transfer the contents to a flat plate. You will observe two clear portions. The central yellow mass is the yolk. It is surrounded by a transparent white jelly-like fluid, called albumen. Albumen and yolk represent the reserve food material in the cytoplasm.

Hen's egg is a single cell. Its different parts have been labelled in the diagram given here.



Cross-section of a hen's egg

The (approximate) sizes, of some of the plant and animal cells, are given in Table 1.

Table 1

	Cell	Size
1.	*Amoeba*	1000 µm
2.	Hen's egg	60 mm
3.	Ostrich egg	170 mm
4.	Green alga, *Chara*	10 cm

The (approximate) sizes, of some cells of human body, are given in Table 2.

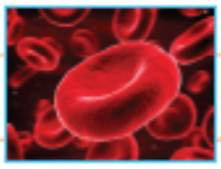
Table 2

	The cell of the human body	Size
1.	Red blood cell	9 µm
2.	Liver cell	20 µm
3.	Human ovum	0.1 mm or 100 µm
4.	Nerve cell	about 1 m

(**Note:** 1 µm = 10^{-6} metres = 10^{-3} millimetres)

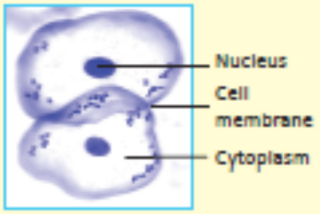
Do You Know ?

The life span of a red blood cell is about 120 days.



Activity 2

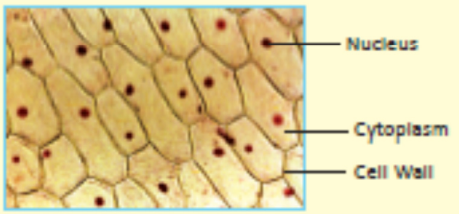
1. To observe animal cells make a temporary mount of cheek cells.
   - Take a clean toothpick.
   - Scratch it gently on the inner side of your cheek.
   - Some frothy material appears on the toothpick.
   - Rub it in the centre of a clean glass slide.
   - Put a drop of methylene blue.
   - Let it stain for a minute.
   - Put a cover slip and observe it under the microscope.



Cheek cells

   You will observe polygonal, isolated cells, or cells in clusters. Observe the darkly stained nucleus in each cell.

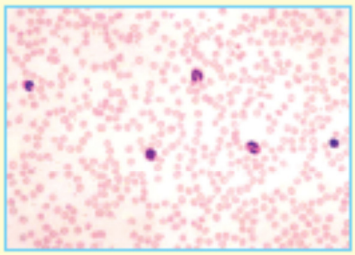
2. Follow the instructions given below to make a slide of onion peel. (Onion peel is the thin membrane-like layer present around fleshy scale leaves of onion.)
   - Put a drop of water on a glass slide.
   - Place a small piece of neatly cut onion peel on it.
   - Put a drop, or two, of saffranin.
   - Stain for a minute.
   - Put a cover slip and observe it under the microscope.



Onion peel showing its cells

   You will see that the cells here are arranged in rows. Observe their boundaries. There is a dark structure in the centre of each cell. It is the nucleus.

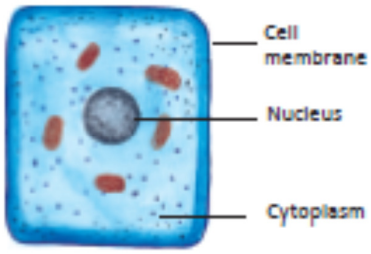
3. To see different types of cells present in blood request your teacher to prepare a slide of human blood. (You may also use a permanent slide of blood to study various types of blood cells.) You can observe red blood cells having their characteristic red colour and their disc shape. You can also observe a few *Amoeba*-like white blood cells, present between the red blood cells.


Microscopic view of blood cells

Parts of a Cell

A cell consists of a living protoplasm surrounded by a **cell membrane**. The protoplasm consists of the **cytoplasm** and the **nucleus**.

Cytoplasm contains a number of structures, which are called **cell organelles**. Organelles are, therefore, structures present within a cell that help it to perform its relevant functions.



Structure of a typical cell

Let us learn more about the different parts of a cell.

Cell Membrane

All living cells are bound by a membrane called the **plasma membrane,** or the **cell membrane**. It surrounds its inner gel-like material called **protoplasm**. The plasma membrane controls the entry and exit of substances as per the requirements of the cell.

The cells of plants, fungi and bacteria have an additional outer covering called the **cell wall**.

The cell wall is an important covering in plant cells; it provides rigidity and protection to the cell against variations in the environment. It also gives a definite shape, size and support to the cell.

Cytoplasm

The portion of the protoplasm, lying inner to the cell membrane but outside the nuclear membrane, is called **cytoplasm** [*kytos* (hollow), *plasma* (liquid)]. It acts as a 'ground substance' for all cell activities. It is made up of carbohydrates, proteins, fats, minerals and vitamins, along with a large proportion of water. All these components work together to provide a unique living nature to the protoplasm.

Nucleus

It is the most important part of the cell. It generally lies in the centre of the cell, however, in some cases, it may also occupy peripheral positions. It controls all the activities of the cell.

The nucleus is a dense structure bound by a **nuclear membrane**. The

protoplasm of the nucleus is called **nucleoplasm**. It has a thread-like network called **chromatin**. When the cell is ready to divide, this chromatin condenses to form thicker, thread-like structures, called **chromosomes**. These chromosomes are the structures responsible for the characters (genes) inherited by one generation from the earlier generations.

Do You Know ?

The number of chromosomes in a cell differs in different organisms. Some are shown below.

Organisms	Chromosome number
Man	46
Dog	78
Pigeon	80
Yeast	32
Wheat	42

Many small living structures are present in the cell. These are equivalent to the organs of the body. Hence, they are named as 'cell organelles'.

Cell Organelles

The main cell organelles are:

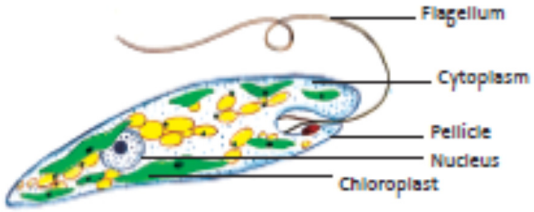
- **Plastids :** These are large cell organelles, characteristic of plant cells. These may contain pigments that provide colour to the cell. The green-coloured plastids are called **chloroplasts**. They manufacture food for green plants by the process of **photosynthesis**. The plastids, associated with the different coloured parts of the plants (like fruits, vegetables and flowers) are called **chromoplasts**. They are responsible for imparting colour (other than green) to the different parts of the plant. Plants also contain some colourless plastids called **leucoplasts**; these provide space to store starch, proteins, oils, etc.
- **Mitochondria:** These are rod-shaped or spherical structures. They are present in large numbers in cells engaged in different physiological activities. They are responsible for cellular respiration and for generation of energy for different activities of life. Hence, they are also called the **powerhouse** of the cell.
- **Endoplasmic Reticulum (ER):** It is a network of membranes. It provides channels for transport of materials in a cell. ER is of two types:

  Rough ER : This type of ER has a rough appearance as it is studded with ribosomes. It plays a vital role in synthesis of proteins.

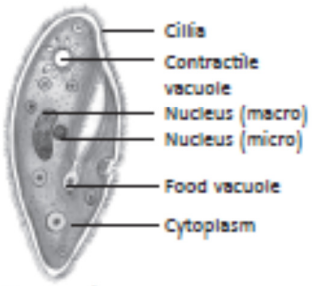
  Smooth ER: This type of ER does not have ribosomes attached to it; it, therefore, has a smooth appearance. It helps in the synthesis of fats.
- **Golgi Complex :** They are sac-like structures stacked one above the

other. They are involved in the processing and packaging of materials produced by the cell.

- **Vacuole:** It appears as an empty space in the cytoplasm. It is generally large in plant cells. It stores excess of water and waste products. In *Amoeba*, food materials are held in its food vacuoles for digestion.
- **Ribosomes:** These are tiny granules present in the cytoplasm and on the rough ER. They help in protein synthesis.
- **Cilia and flagella:** Some cells have these small extensions on their cell membrane. They help in locomotion and collection of food. Unicellular organisms, like *Paramoecium*, have numerous cilia while *Euglena* has a single flagellum.



Euglena



Paramoecium

All these cell organelles work together to perform different functions of the cell.

Levels of Organisation in an Organism

In unicellular organisms, like *Amoeba*, a single cell performs all the necessary functions. It captures and digests food, respires, excretes, grows and reproduces.

Multicellular organisms have cells that are specialised to perform specific functions. A group of cells, performing a specialised function, forms a **tissue** (for example, nervous tissue). A group of tissues, performing a specific function, forms an **organ** (for example, kidney). A number of such organs work together to form an **organ system** (for example, digestive system).

Do You Know ?

The following organ systems work in the human body.

(i) Digestive	(ii) Respiratory	(iii) Circulatory	(iv) Excretory	(v) Skeletal
(vi) Muscular	(vii) Nervous	(viii) Reproductive	(ix) Endocrine	(x) Integumentary

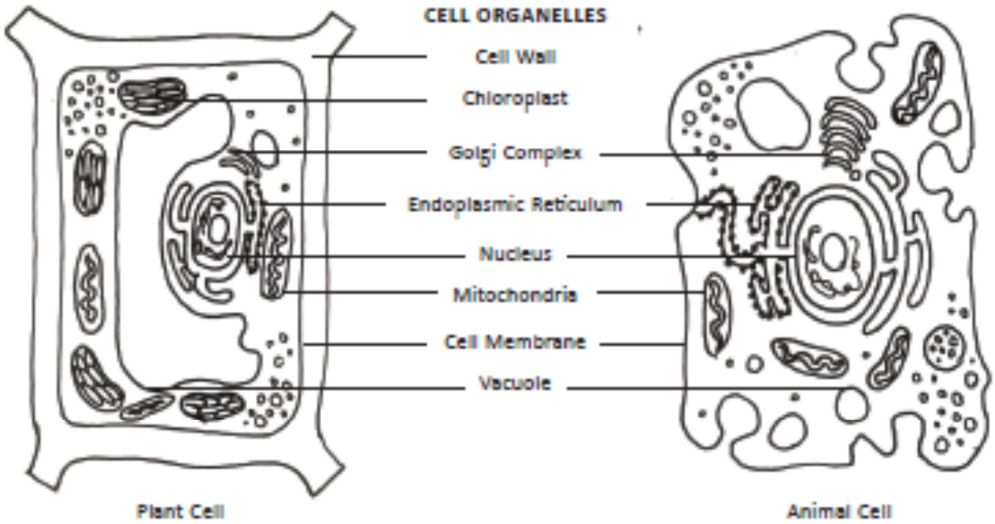
All the cells have some common features. However, they can appear different in different parts of the organism. For example, the blood and liver cells (in animals), the root or leave cells (in plants) have different appearances.

The plant and animal cells, however, have some major differences between them. Let us now, compare the features of the plant and animal cells.

Comparison Between Plant and Animal Cells

Although all living cells have certain common features, detailed studies reveal some major differences between plant and animal cells.

Plant cells generally have a definite shape due to a rigid cell wall around them. In comparison, animal cells have a cell membrane as their outer cover. This provides flexibility to animal cells; hence they can show a large variation in their shapes. Moreover, plant cells have plastids; these are absent in animal cells. Plant cells generally have large vacuoles; animal cells, on the other hand, either lack vacuoles, or have very small vacuoles.



The main points of difference, between a plant cell and an animal cell, have been summarised in the table given below.

Difference between a Plant and an Animal Cell

Components/Characters	Plant Cell	Animal Cell
Shape	Fixed	Irregular/Not fixed
Cell Wall	Present	Absent
Plastids	Present	Absent
Vacuoles	One large vacuole is present	Vacuoles are either absent, or are present only as small vacuoles.

[**Note:** Cell organelles, other than the ones shown in the diagrams on the previous page, are also present in the cells. However, they will be discussed in higher classes.

Keywords

cell	basic structural and functional unit of life.
cell membrane	a thin membrane that surrounds the protoplasm of every cell.
cell organelles	a specialised sub-unit, within a cell, that has a specific function.
chromosomes	thread-like structures found in the nucleus; responsible for the inheritance of characters.
cytoplasm	portion of protoplasm, lying between the cell membrane and the nuclear membrane.
cilia and flagella	extensions on the cell membrane, these help in locomotion and procurement of food in organisms like *Amoeba* and *Paramoecium*.
endoplasmic reticulum	network of membranes which provides channels for transport of materials in the cell and helps in synthesis of proteins.
genes	unit of inheritance which gets transferred from one generation to the next.
golgi complex	sac-like structures; these help in processing and packaging of materials produced by the cell.
mitochondria	rod-shaped structures inside a cell; these help in cellular respiration and production of energy.
nucleus	a specialised structure in the cells, bound by the nuclear membrane; responsible for controlling all cellular activities.

plastids	cell organelles found in plant cells. These may contain pigments which help in photosynthesis and are responsible for imparting colour to fruits, vegetables and flowers.
protoplasm	gel-like living matter present inside the cell membrane.
ribosomes	tiny granular structures found in the cytoplasm and on the endoplasmic reticulum; they help in protein synthesis.
tissue	group of cells performing a specialised function.
vacuole	sac-like membrane bound structures in cells; used for storing various materials.

You Must Know

1. Cell is the basic structural and functional unit of all living organisms.
2. Living organisms show variation in their cell number, shape and size.
3. Unicellular organisms, like *Amoeba*, are made up of a single cell; multicellular-organisms, like a mango tree or a parrot, are made up of many cells. Cell sizes may vary from (nearly) 0.1 micron (*Mycoplasma*) to 170 mm (Ostrich egg) in diameter.
4. A cell consists of living matter, called protoplasm, surrounded by a cell membrane. Plants, fungi and bacteria have an additional cover, known as the cell wall, outside their cell membrane.
5. Protoplasm consists of cytoplasm and a nucleus.
6. The nucleus controls all the activities of the cell. The cytoplasm contains many cell organelles; these perform various functions in a cell.
7. Some of the cell organelles, and their functions, are as follows:

   Mitochondria are responsible for respiration; green coloured plastids, or chloroplasts are the site of photosynthesis; golgi complex processes materials produced by the cell; vacuoles store excess water and waste; ribosomes help in protein synthesis, and cilia and flagella help in locomotion.
8. A tissue is a group of cells performing a specialised function.
9. An organ is formed by a group of tissues that perform a specialised function. When a number of organs work together, they form an organ system (for example, digestive system).
10. Plant and animal cells show some major differences. Plant cells possess a cell wall and plastids; these are not found in an animal cell. Also, plant cells contain large vacuoles; the vacuoles are either absent in animal cells, or if present, have a small size only.

Something To Know

A. Fill in the blanks.

1. All living organisms are made up of _____ .
2. The _____ provides rigidity and protection to the plant cell.
3. All cellular activities are controlled by the _____ .
4. The _____ is known as the powerhouse of the cell.
5. _____ is a group of cells performing a specific function.
6. It was _____ who observed cells for the first time.

B. Match the following.

1.	Golgi Complex	(a)	genes
2.	Ribosomes	(b)	cork
3.	Chromosomes	(c)	chloroplasts
4.	Dead cells	(d)	packaging centre
5.	Photosynthesis	(e)	protein synthesis

C. Tick (✓) the correct option.

1. The cell organelle, which acts as a storage bag for a cell, is known as the—

   ☐ choloroplast ☐ chromoplast

   ☐ mitochondria ☐ vacuole

2. Hen's egg is—

   ☐ a cell organelle ☐ a tissue

   ☐ a single cell ☐ an organ

3. The nucleus is separated from the cytoplasm by the—

   ☐ cytoplasm ☐ nuclear membrane

   ☐ cell membrane ☐ protoplasm

4. Which of the following will not be found in an egg cell, human liver cell and an *Amoeba*?

- ☐ ribosomes
- ☐ cell membrane
- ☐ mitochondria
- ☐ cell wall

5. Which of the following represents the correct sequence?

- ☐ tissue → cell → organ → organ system
- ☐ organ → tissue → organ system → cell
- ☐ cell → organ → tissue → organ system
- ☐ cell → tissue → organ → organ system

6. Which, amongst the following pairs, can be found only in a plant cell but not in an animal cell?

- ☐ cell wall and plastids
- ☐ plastids and cilia
- ☐ cell wall and cell membrane
- ☐ plastids and mitochondria

D. Answer the following questions in brief.

1. List the (main) factors that determine the shape of a cell.
2. Distinguish between unicellular and multicellular organisms. Give two examples of each.
3. Give reasons for the following:

   (a) The cell is called the structural and functional unit of life.

   (b) Plant cells are more rigid than the animal cells.
4. Which cell organelle is known as the 'Powerhouse of the cell'? Why is it so called?
5. Name the cell organelles responsible for imparting colour to the leaves and fruits of a plant.
6. What are cilia and flagella? Write one similarity and one dissimilarity between the two of them.

7. If onion peel cells and cheek cells are observed through a microscope, state the two major differences that the observer is likely to find.

8. Classify the following into cells, tissue and organ.

   skin, fat cell, RBC, blood, ear, muscle

Cell	Tissue	Organ

E. Answer the following questions.

1. "All cells in an organism do not have the same shape."

   Justify the above statement by drawing at least three different cell types found in human beings.

2. Where, and how, are chromosomes formed? State their significance.

3. With the help of well labelled diagrams, highlight three differences between a plant cell and an animal cell.

4. Write the functions performed by the following cell organelles.

   (a) Endoplasmic Reticulum

   (b) Golgi Complex

   (c) Nucleus

   (d) Chromoplasts

   (e) Vacuoles

   (f) Mitochondria

5. The cell membrane is a very important component of a cell. How is damage to the cell membrane likely to impact the functions of the cell?

6. Define the term 'cell', for plants/animals. Name the different organelles that make a cell. Explain why none of these is called the structural and functional unit of life.

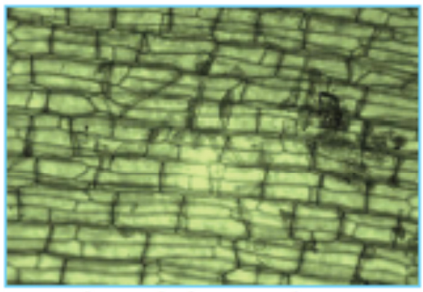
Value Based Question

The biology teacher started her lesson on the structure and functions of the animal/plant cells by talking about a cricket team. She told her students that a team wins its matches only when all its members 'work as a team' and do their assigned roles in an efficient, sincere and responsible way. She then went on to say that the animal/plant cells also work properly as their different parts work as a 'team' and do their specific funtions. She advised her students to imbibe the 'team spirit' and do their specific functions as responsible members of any 'team' they may be a part of.

1. Name two of the values that the teacher spoke of in her class.
2. Have a group discussion in which students suggest how, 'working as a team' can improve the 'overall working' of a home or a school.
3. Have a 'play' in the class in which a group of students give themselves names corresponding to the different cell organelles of a plant/animal cell. Each 'name' has to then explain her/his role in the working of the cell.

Something To Do

1. ***Hydrilla* is an aquatic plant that can be seen in lakes and ponds. It has small, thin leaves. Take a *Hydrilla* leaf and place it on a glass slide. Observe it under a microscope. Note your observations.**



Microscopic view of *Hydrilla* leaf

2. **Divide the class into four groups of students. Each group will prepare a short report on the topics/questions given below. Two students from each group may then present the report to the whole class.**

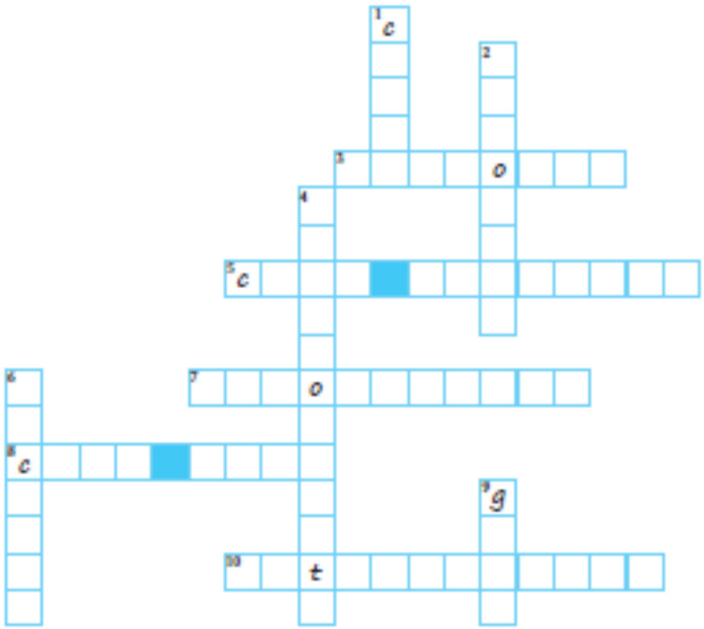
   (a) When were cells discovered? Were they discovered before or after the invention of the compound microscope?

   (b) Does the size and number of cells depend upon the size of the organism?

   (c) List the functions of cell organelles and also mention the organ/organ system, that the organelle is similar to, in the human body.

   (d) List the advantages of having a cell wall as the outermost boundary in cells.

3. **Complete the crossword puzzle given below.**



Across →

3. storage structures of a cell.
5. regulates the entry and exit of materials in and out of the cell.
7. impart colour to flowers and fruits.
8. provides shape and rigidity to a plant cell.
10. powerhouse of the cell.

Down ↓

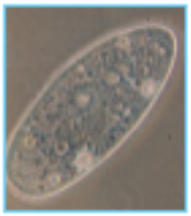
1. help *Paramoecium* in locomotion.
2. structure that helps in protein synthesis.
4. help plant cells in converting solar energy into usable form.
6. control centre of the cell.
9. responsible for transferring traits from one generation to next.

CHAPTER

2 Microorganisms: Friends or Foes

We see a large number of plants and animals around us. Besides these, there are a large number of small organisms which cannot be seen with our unaided eye. However, they can be easily observed through the microscope. These living organisms, that are invisible to the naked eye but are visible under the microscope, are called **microorganisms**; their study is known as **microbiology**. Microorganisms include viruses and single-celled organisms like bacteria, yeast, protozoans and algae. Each of these groups of microorganisms includes some harmful organisms (foes) and some useful organisms (friends).

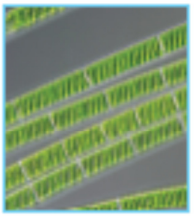
Bacteria



Paramoecium

Euglena

Amoeba



Spirogyra

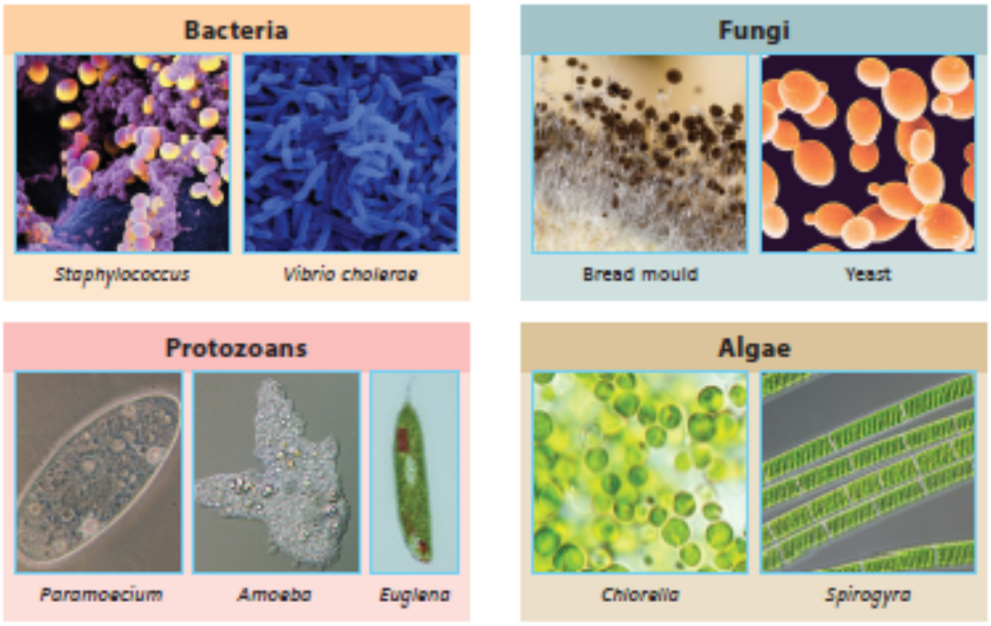
The discovery of microorganisms was possible due to the invention of the microscope. Anton von Leeuwenhoek was the first person to establish the existence of bacteria. Now, with the help of modern microscopes and other new techniques, we have come to know a lot more about microorganisms.

Types of Microorganisms

Microorganisms are the oldest forms of life on earth. Certain varieties of microorganisms have existed for millions of years. They affect us in many ways.

On the basis of their cell structure, microorganisms can be divided into four groups. These groups are: bacteria, fungi, protozoa and (some) algae.

Some of these organisms have been shown in the following pictures.

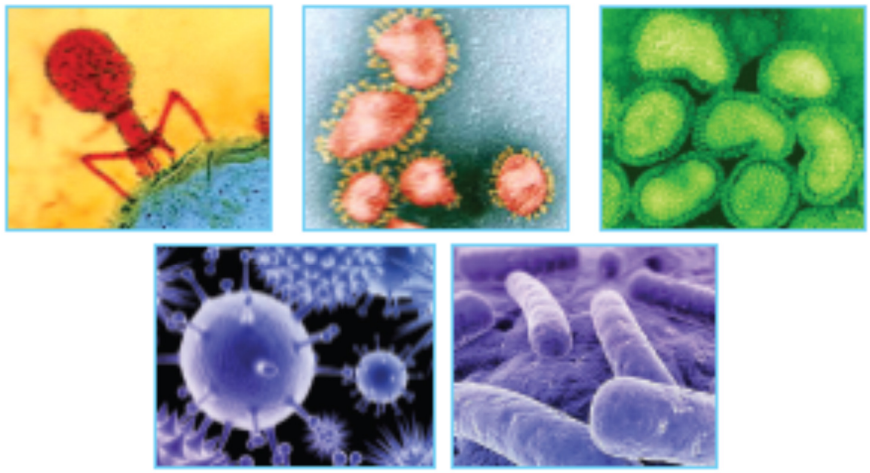


Types of Microorganisms

Viruses are Unique

Viruses are unique; they can exhibit the characteristics of both living organisms and non-living things. When they are floating in air, or are settled on a door

knob, they are non-living like, say, salt and sugar. In fact, they can be crystallised and stored in jars for years. However, when they come in contact with a suitable plant, animal or bacteria, they show the characteristics of living organisms. They infect the cell and quickly multiply inside it. Viruses are, therefore, regarded as being on the borderline between living and non-living entities.



Viruses of different shapes

Where do Microorganisms Live?

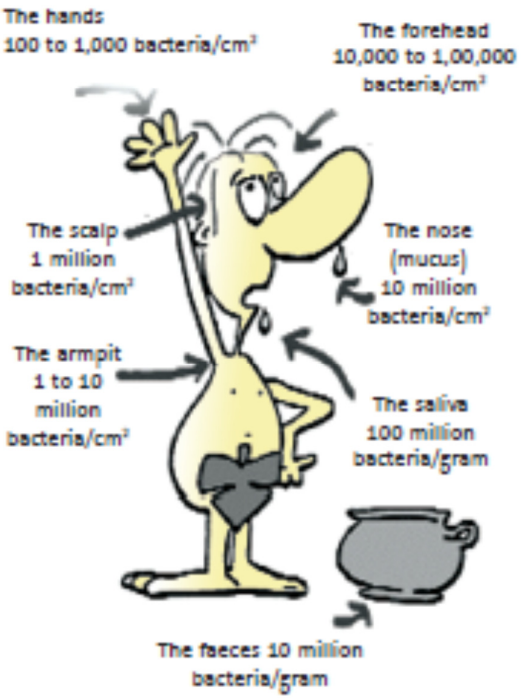
Microorganisms are found in almost all kinds of environment: in ice-cold water, in hot springs, in dry, marshy or saline areas. Some of them need oxygen for their growth while others do not. They are found in soil, on the ocean floor, high in the atmosphere and deep inside rocks within the earth's crust. Microorganisms are also found in the human body and in the bodies of other plants and animals. Our mouth, throat, nose and the alimentary canal are all inhabited by a large number of microorganisms. Thus, microorganisms are found everywhere.

Do You Know?

- Extremophiles are microorganisms which have adapted themselves so that they can survive, and even thrive, in conditions that are normally fatal to most life forms.
- Hyperthermophiles are organisms that can thrive even at temperatures between 80°C–122°C, such as those found in hydrothermal systems.
- Hypoliths live inside rocks in cold deserts.
- Cryophiles grow better at temperatures of 15°C or lower; common in cold soils and polar ice-cold ocean waters.

Role of Microorganisms in Our Life

Microorganisms play an important role in our lives as well as in the overall environment. They help in the decomposition process and in maintaining the biogeochemical cycles (like the carbon and nitrogen cycles). They are useful and beneficial for mankind in many ways. However, some of them are harmful as they spoil our food and cause diseases.



Microbial Population in the Human Body

Under normal conditions, our bodies house a large population of microorganisms; they are, however, kept in balance and are usually harmless. These microorganisms are important for our body; they form an essential system that helps our body.

Microorganisms as Our Friends

Microorganisms are friendly to us in many ways.

Uses in Food

Curd and cheese formation

Lactobacillus is a bacterium that helps in the formation of curd. At favourable temperatures, it multiplies in milk and converts it into curd. Some bacteria and fungi are also involved in the making of cheese.

Fermentation process in bakery

Fungi, like yeast, reproduce rapidly and produce carbon dioxide. This gas, when trapped in dough, or batter (used for *idlies*, *dosas*), causes it to increase in volume and makes it fluffy and soft. This is known as **fermentation**.

Alcoholic beverages

Many microorganisms are used in the manufacture of alcohol, wine and acetic acid. Fungi, like yeast, convert natural sugars, present in cereals and fruits, into alcohol; this alcohol is then used to make alcoholic beverages. Acetic acid, commonly known as *vinegar*, is also produced by a similar process.

Activity 1

Take five beakers and label them as A, B, C, D and E. You may label them with a marker pen. Put 200 ml of milk in each of the five beakers. Heat the milk in the first four beakers to the temperatures mentioned in the table given below. Add a teaspoonful of curd in each beaker and cover each beaker with an inverted petridish. Put the beakers in separate cardboard boxes, or insulated containers. You may wrap each beaker in a thick towel to help maintain their respective temperatures. Put the fifth beaker in the refrigerator, again after mixing a teaspoonful of curd in it. Leave the beakers undisturbed for 3-4 hours. Record your observations in the given table.

Beaker	Temperature at the time of mixing curd with milk	Observation (has the milk been converted to curd) Yes/No
A	40°C	
B	50°C	
C	60°C	
D	100°C	
E (kept in the refrigerator)	5-8°C	

Most microorganisms exhibit maximum growth in the temperature range 30°C-45°C. It is for this reason that we store perishable materials in the refrigerator.

Milk tastes sweetish because of the presence of a sugar called **lactose**. *Lactobacillus* converts this lactose (of the milk) into lactic acid in curd. The longer you leave curd at room temperature, the more sour it tastes; this is because of increased growth of bacteria and the resulting production of more lactic acid.

Sewage Treatment

Some bacteria are used in the biological treatment of sewage and industrial waste, called **effluent**. This process is known as **bioaugmentation**.

Importance in Human Health

1. Many microorganisms, present in the alimentary canal of some animals (like cows), help in digestion and absorption of food. The bacteria, present in our large intestine, help in bowel movement.

2. Microorganisms are also used in production of antibiotics. **Antibiotics** are chemicals that inhibit the growth of (other) harmful microorganisms by affecting their life processes. For example, penicillin is an antibiotic obtained from a fungus, *Penicillium notatum*. Streptomycin, tetracycline and erythromycin are some antibiotics obtained from fungi and bacteria.

   Antibiotics are extremely effective in treatment of various microbial infections/diseases, like, tuberculosis, cholera, etc. However, antibiotics should be taken only on the advice of a qualified doctor and that too only in the prescribed dosage and for the prescribed duration. Not completing the prescribed course may make them ineffective when used in future. If they are taken when not really required, they may kill some of the useful bacteria in the body.

3. When microorganisms, like bacteria or viruses, enter our body, they are recognised by special kind of blood cells. These cells get stimulated to produce **antibodies**. Antibodies identify and destroy such disease causing organisms.

   During this process the body 'remembers' the type of microorganisms; if the same microorganism enters the body again, it gets recognised and destroyed much faster. This is called 'immunity'. **Immunity** is, therefore, the natural ability of an organism to have an inbuilt mechanism to resist, and destroy, the infection that some microorganism may cause.

   Immunity through Vaccination

   Vaccination is an important way to build immunity. A vaccine produces immunity to a disease by stimulating the production of antibodies. **Vaccines** are suspensions of killed, or weakened microorganisms (or products, or derivatives, of such microorganisms). The most common method of administering vaccines is by inoculation; however, some vaccines are given orally also.

Do You Know?

Edward Jenner was an English doctor who pioneered the vaccination process. Jenner's discovery in 1796 – inoculation with cowpox gave immunity to smallpox – was an immense medical breakthrough and has saved countless lives. Smallpox has now been eradicated from the world.

- ## Microorganisms in Agriculture

Some bacteria, and blue-green algae (cyanobacteria), are able to 'fix' (for use by plants) atmospheric nitrogen into usable forms of salts of nitrogen. They are called **biological nitrogen fixers**. *Rhizobium* lives in symbiotic association in the root nodules of leguminous plants and enriches the soil with nitrogen compounds. Some cyanobacteria do the same in rice fields and in association with the roots of *Cycas* plant.

Do You Know?

Probiotics (dietary supplements of live bacteria or yeasts) can help prevent, and treat diseases through a number of mechanisms. One way is by interacting directly with the disease-causing microbes making it harder for them to cause disease. An example of this is the ingestion of probiotic bacteria to prevent, or to treat, diarrhoea. These organisms help reinforce the natural bacterial barrier that exists on the lining of the digestive tract; they thus, provide additional protection against pathogenic organisms that can cause diarrhoea.

- ## Use in Energy/Fuel Production

Many microorganisms produce ethanol by fermentation of sugars and produce methane in the biogas reactors. Both ethanol and methane are used as fuel for production of energy.

- ## Cleaning the Environment

When a plant or animal dies, it leaves behind nutrients and energy in the organic material that formed its body structure. Decomposers eventually convert all such organic matter into carbon dioxide and nutrients. These nutrients (like nitrogen, phosphorous, magnesium, etc.) become a part of the soil. This process eventually replenishes nutrients back to the ecosystem, thereby, allowing the plants to grow.

Bacteria and fungi are some of the common decomposers.

Microorganisms – The Foes

Some microorganisms are harmful in different ways. They cause diseases in human beings, plants and animals. Such disease causing organisms are known as **pathogens**. Some microorganisms also cause spoilage of food. Some grow on leather and clothes and have an adverse affect on their quality.

Bacteria also grow on food particles that may get lodged in between our teeth. Acids produced by these bacteria, corrode the tooth enamel and cause cavities. Many microorganisms grow in water bodies and decrease their oxygen level. This is harmful to the other organisms living in these water bodies.

Microorganisms causing diseases in Humans

During our lifetime, we come in contact with many microorganisms. Some of these microorganisms leave undesirable effects on our body. They invade our body and multiply inside it. They release some harmful materials, called **toxins**, in our body. These toxins adversely affect our body and can make us suffer from diseases.

Mosquito — *Aedes*

Diseases, which can spread from an infected person to a healthy person, are called **communicable diseases**. Communicable diseases spread from one person to another through air, water, food, physical contact or insects. In diseases, like tuberculosis and pneumonia, when a patient sneezes or coughs, small droplets carrying germs, are released in the air. These germs, if inhaled, can infect a healthy person. The consumption of contaminated food can result in diseases like cholera, typhoid or hepatitis. Different types of insects can also spread various diseases. For example, malaria is spread by the female *Anopheles* mosquito, dengue by *Aedes* mosquito and plague by rat flea. Diseases, like common cold and conjunctivitis, can spread through direct, or indirect, contact with a patient.

Given below is information about some common human diseases and the way they get transmitted.

Infectious Diseases Caused by Microorganisms

Microorganisms	Diseases Caused
Bacteria	Tuberculosis, diphtheria, cholera, tetanus, typhoid
Viruses	Common cold, influenza, mumps, polio, chickenpox, AIDS
Fungi	Ringworm, athlete's foot
Protozoans	Malaria, amoebic dysentery, sleeping sickness

Modes of Transmission of Pathogens

Modes of Transmission	Related Diseases
Air (actions like sneezing, coughing)	Tuberculosis, common cold, influenza, swine flu
Water (using contaminated water)	Typhoid, amoebic dysentery, cholera
Soil (consuming food items that are not washed/cleaned properly; wounds)	Tetanus
Animals (direct contact (e.g. being bitten by a rabid dog); through vectors that spread disease)	Rabies, malaria

Coughing and sneezing

Dog bite

Mosquito bite

Prevention of Diseases

Various ways that can help in preventing diseases are given in the following table.

Methods of prevention	Diseases that may be prevented
Vaccination	Tuberculosis, polio, mumps, tetanus
Using mosquito nets and mosquito repellents, controlling mosquito population	Malaria, dengue
Proper disposal of waste and using disinfected water	Cholera
Maintaining good personal hygiene	Ringworm and athlete's foot

Microorganisms Causing Diseases in Animals

Microorganisms also cause diseases in animals. You must have heard about rabies (a disease that affects animals like dogs), and foot and mouth disease (that affects cattle). Such diseases may affect domestic animals. Pets and human beings may also get infected if they come in contact with the infected animals.

The following table tells us about some diseases caused by the microorganisms in animals.

Name of the disease	Microorganisms that cause the disease	Animals infected
Anthrax	Bacteria	Cattle
Foot and mouth disease	Virus	Cattle
Rabies	Virus	Dogs, monkeys
Tuberculosis	Bacteria	Cattle, poultry
Ringworm	Fungi	Cattle, poultry
Aspergillosis	Fungi	Poultry
Canine distemper	Virus	Dogs

Microorganisms Causing Diseases in Plants

You must have observed some plants with wrinkled leaves, or plants and trees having an abnormal branching pattern or having abnormal fruits. These plants may have been affected by a pathogen. Most plant diseases are caused by fungi, bacteria and viruses. These microorganisms may reduce crop yield and, at times, cause total destruction of the crops. The Irish Famine in 1740-41 was caused because of complete destruction of the potato crop due to the disease, 'late blight', caused by a fungus.

The following table gives information about some common plant diseases that are caused by different microorganisms.

Microorganism	Related Plant disease
Virus	Tobacco mosaic virus (appearance of uneven spots and discolouration on the leaves)
Bacteria	Citrus canker (lesions on leaves, stems and fruit, with raised brown water soaked margins)
Fungus	Rust of wheat (diseased plants show rust-coloured orange patches on the infected plant parts)
Fungus	Smut of rice (smut balls in grains that become greenish black; then burst and infect other grains)
Fungus	Red rot of sugarcane (the infected stems have a dull red colour interrupted by occasional whitish patches across the stalk, and/or elongated red lesions on the midribs of leaves)


Tobacco mosaic


Citrus canker


Wheat rust


Smut of rice

Plant diseases caused by microorganisms

Food Poisoning

Food poisoning, as the name suggests, is a disease that results from the consumption of contaminated food. Symptoms of food poisoning are vomitting, nausea, severe pain in the abdominal region, diarrhoea and dehydration. It may also cause weakness, fatigue and damage to the nervous system.

These symptoms arise due to the presence of bacteria, or other microbes, in food. They may also be due to ingestion of toxins contained in food (including those produced by bacteria). Some bacteria, like *Clostridium* and *Staphylococci,* and fungi, like *Aspergillus*, cause food poisoning.

Food must be properly prepared and stored to prevent food poisoning. Food poisoning can occur when food is left unrefrigerated for long periods of time. This often happens at picnics and large parties. Sometimes, mishandling also causes food poisoning. Persons, who handle or prepare food, should wash their hands to prevent contamination of food.

Food Preservation

Food preservation is the process of treating and handling food so as to stop, or greatly slow down, spoilage (loss of quality, edibility or nutritive value) caused, or accelerated, by microorganisms.

Preservation usually involves preventing the growth of bacteria, fungi and other microorganisms, as well as retarding the oxidation of fats which cause **rancidity**. (Rancidity means having disagreeable odour, or taste due to decomposition of oils or fats.)

Some preservation methods require the food to be sealed after treatment to prevent recontamination with microbes; other methods, such as drying, allow food to be stored without any special containment for long periods.

A brief description of some of the methods used for 'food preservation' is given below.

1. **Drying:** This method reduces water content sufficiently and, thereby, prevents, or delays, bacterial growth. Drying also reduces weight, making food more portable. Some common food stuffs that are preserved by drying are: apples, pears, bananas, mangoes, papaya, apricot and coconut. Drying is also the normal means of preservation for cereals (grains), such as wheat, maize, oats, barley, rice, millet and rye.

2. **Preservation using sugar and salt:** Sugar and salt reduce the water content and make it unavailable for the growth of microorganisms. As a result, the food gets preserved. Meat and fish are covered with salt to check bacterial growth. Some pickles are also preserved by the addition of specific quantities of salt. Jams and jellies are usually preserved by addition of sugar.

Pickling for the preservation of food

3. **Preservation using acetic acid:** Acetic acid, in the form of *vinegar*, is used in the manufacture of several pickled products. *Vinegar* also stops the growth of microorganisms. Citric acid and phosphoric acid are also used, in carbonated beverages and fruit drinks, for both flavouring and preservation.

4. **Preservation using chemical preservatives:** Chemical preservatives, like sodium benzoate and potassium metabisulfite are used to preserve jams, jellies and pickles. These chemicals inhibit the growth of microorganisms.

5. **Heat and cold treatments:** Some food items, like milk, are usually boiled, before their use, or storage. Boiling kills many microorganisms.

   Several food stuffs are also stored in the refrigerator; the low temperature inhibits the growth of microorganisms.

6. **Pasteurisation:** Pasteurisation is 'pressurised heating' for a short time; it may be considered as a mild form of heat treatment. The temperature, used during pasteurisation, is below 212°F (100°C). Milk is pasteurised to destroy microorganisms. However, there are many more heat-resistant organisms in it that only get reduced in number. Hence, pasteurised milk needs to be stored under refrigeration to keep bacterial growth in check.

   In addition to destroying some microorganisms, pasteurisation also inactivates some enzymes; that, at times, can be a disadvantage.

7. **Vacuum packing:** Vacuum packing stores food in a vacuum environment; usually an air-tight bag or bottle. The vaccum environment deprives bacteria of the oxygen needed for their survival; it, therefore, slows 'spoiling'. Vacuum packing is commonly used for storing nuts; it helps to reduce their loss of flavour caused by their oxidation.

Vacuum packed food

8. **Canning:** It involves cooking food and sealing it in sterile cans or jars; this is followed by boiling the containers to kill or weaken any remaining bacteria (a form of sterilisation). However, food preserved by canning, or bottling, is at immediate risk of spoilage once the can or bottle has been opened.

   At times, the cans or jars get damaged and show puffing or enlargement. This indicates spoilage of food; the food, in such puffed or enlarged containers, should never be consumed.

9. **Freezing:** When foods are kept at below freezing temperatures, most chemical changes take place at such a reduced rate that only minor changes are noticeable even after long periods of storage. However, microorganisms are generally more resistant to cold than to heat. Although some are killed by freezing, most bacterial spores and a large number of organisms survive and get revitalised when the food is thawed.

Keywords

antibiotic	chemicals produced by microorganisms that kill, or inhibit, the growth of other harmful microorganisms.
antibody	proteins produced by special kind of blood cells to identify and destroy microbes, like bacteria, viruses, etc.
bioaugmentation	using bacteria in treatment of sewage and industrial waste.
biogeochemical cycle	a pathway for cycling of nutrients between the abiotic (air, water, soil) and biotic (plants, animals, microorganisms) components of the ecosystem.
communicable disease	a disease which spreads from an infected person to a healthy person.
fermentation	an anaerobic process through which sugars/carbohydrates are converted into alcohol/acids and carbon dioxide by microorganisms, like yeast.
germs/ pathogens	disease causing microorganisms.
immunity	ability of an organism to resist an infection.
microbiology	the study of microorganisms.
pasteurisation	a method of preserving milk by heating it to 72°C followed by quick cooling.

rancid	an unpleasant taste or odour (of food containing fats and oils) caused by chemical changes, or decomposition.
toxin	a poisonous substance produced by living organisms.
vaccine	a suspension of killed, or weakened, microorganisms, administered to increase protection against a disease, i.e. to bolster immunity.

You Must Know

1. Living organisms, which are not visible to the unaided eye and can be seen only through a microscope, are known as microorganisms. Microbiology is the study of microorganisms.
2. Depending upon their cell structure, microorganisms are classified into Bacteria, Fungi, Protozoa, (some) Algae.
3. Viruses are microscopic bodies which can reproduce only inside the cells of some host organisms, like a bacterium, plant or animal; however, they can be crystallised, (like salt and sugar), when they are outside a living organism.
4. Microorganisms are found everywhere: in air, in water, in rocks within the earth's crust, inside human beings, plant and animal bodies, and even in cold deserts and hot springs.
5. Some microorganisms are useful to us in many ways: they help us in preparing foods, like curd, cheese and alcoholic beverages; they are useful in treatment of sewage, in energy production, and in cleaning of the environment. They also help in production of antibiotics and vaccines. Some of them even enrich the soil and help in agriculture.
6. Some other microorganisms are quite harmful. They (pathogens) cause diseases in humans, plants and animals. Some of them cause spoilage of clothes and leather. Some microorganisms grow in water bodies and decrease their oxygen levels; they thus, cause harm to other organisms living therein.
7. Mishandling, and improper storage of food, causes microbes to contaminate it and produce toxins in it. Consuming such food causes food poisoning; this can result in vomitting, nausea, dehydration; it may even damage the nervous system.
8. Food preservation is the process of giving an appropriate physical, or chemical treatment to food in order to prevent, or slow down, its spoilage.
9. Some of the methods used to preserve food are: Drying, Use of salt and sugar, Use of preservatives, Pasteurisation, Canning, Freezing and Vacuum packing.

Something To Know

A. Fill in the blanks.

1. The study of microorganisms is known as _____ .
2. The bread dough rises because of the production of _____ .
3. _____ are the microorganisms that cause diseases.
4. _____ and _____ are two chemical preservatives.
5. _____ is a fungus that causes food poisoning.

B. Match the following.

1.	Ringworm	(a)	food poisoning
2.	Clostridium	(b)	biogas reactor
3.	Anthrax	(c)	fungus
4.	Lactobacillus	(d)	cattle
5.	Methane	(e)	curd

C. Tick (✓) the correct option.

1. The process, that helps milk to last longer but does not kill all microbes present in it, is known as—
   - ☐ vaccination
   - ☐ bioaugmentation
   - ☐ pasteurisation
   - ☐ fermentation
2. Rust of wheat is caused by—
   - ☐ fungus
   - ☐ bacteria
   - ☐ virus
   - ☐ protozoa
3. Medicines, containing killed or weakened pathogens, and used to prevent infectious diseases, are called—
   - ☐ disinfectants
   - ☐ antibodies
   - ☐ antibiotics
   - ☐ vaccines

4. Which of the following reproduces only inside a host cell?

☐ virus ☐ alga

☐ protozoan ☐ bacteria

5. A disease, caused by a virus, and spread by an insect, is—

☐ polio ☐ dengue

☐ rabies ☐ mumps

D. Answer the following questions in brief.

1. Why are viruses considered as being at the 'borderline' between living and non-living things?
2. What is fermentation? How is this process useful in the food and beverage industries?
3. 'In the absence of microorganisms the earth would become a heap of dead plants and animals.' Justify this statement.
4. Munna observes that the yield of wheat, growing in his field, has reduced in the current year. He gets the soil tested and the report confirms the deficiency of one particular nutrient. Accordingly, he is advised to grow peas after harvesting wheat.

   (i) Name the nutrient found insufficient.
   (ii) How do you think growing peas will help in replenishing the soil?
5. Why does it take less time to prepare curd in summers as compared to preparing it in winters?
6. How are the following diseases transmitted:

   (a) Malaria (b) Common cold (c) Tetanus (d) Typhoid
7. What are antibiotics? How are they produced? Give two examples of these.

E. Answer the following questions.

1. 'Microorganisms are very useful in manufacturing different food items'.

   Explain the above statement.

2. Give reasons for the following.

   (a) Yeast is added to the batter used to make '*dosas*'.

   (b) It is important to brush one's teeth before going to bed.

   (c) Reducing the quantity of salt in pickle can cause it to go bad soon.

   (d) Foods, from puffed or enlarged cans, should not be consumed.

   (e) When using frozen food items one should take out only the required quantities; thawed food should never be refrozen.

3. How do communicable diseases spread? Suggest ways to prevent the following diseases:

   (a) Tuberculosis (b) Athlete's foot (c) Cholera

4. What are pathogens? Name two pathogens each that cause diseases in (i) plants and (ii) animals.

5. Mohan bought *samosas* from a road-side vendor and ate it. After sometime he felt nausea and started vomitting. He had severe pain in the abdominal region and suffered from diarrhoea. What could be the reason for his condition? What is it called? How could it be prevented?

6. Which diseases the following children are most likely to suffer from?

   (a) Ramu drinks water from a nearby lake. This water is neither boiled nor disinfected.

   (b) Ashu does not take a bath everyday. He maintains poor personal hygiene.

   (c) Pulkit has been bitten by an infected street dog.

   (d) Sohan is living in an area where the population is very high. He does not use a mosquito net (while sleeping), or insect-repellent creams (while playing in the open).

   (e) Mohan was with two of his friends who were coughing and sneezing (they were suffering from common cold). One of these friends did not keep a handkerchief in front of his mouth while coughing and sneezing.

Value Based Question

Soham and his wife are overjoyed after the birth of their daughter. They share their joy with their relatives and friends. Soham's wife decides to take their daughter for her first round of vaccinations. However, Soham's mother does not like the idea of putting the child through the discomfort of 'injections' Soham explains to his mother, in a polite and patient way, that this slight discomfort and pain is necessary as it shall ensure that the child lives a long and healthy life.

1. Was Soham right in supporting his wife's decision? List any two values that are demonstrated by his behaviour.
2. How does vaccination help in providing immunity?
3. Find out the names of any five diseases that can be prevented by timely vaccination of children.

Something To Do

1. **Visit a nearby health centre, or contact a doctor. Find out the names of diseases for which vaccinations are available and the age at which they have to be given to the child.**
2. **Genetic engineering is a branch of science in which microorganisms are being extensively used. Try to find out the role of microorganisms in genetic engineering.**
3. **Visit an animal health centre or a veterinarian (a physician for animals). Collect information about various vaccinations that are available for domestic animals (like cows) and pets (like dogs).**
4. **Write how the following methods of preservation work. Also, add two examples of foods that can be preserved by each of these methods.**

Method	How it works	Examples (of food preserved)
Pickling		
Ozone Treatment		

Method	How it works	Examples (of food preserved)
Drying		
Adding Preservatives		
Canning		
Vacuum packing		
Freezing		
Radiation		
Salting		

5. **Find out about the temperatures at which the following occur/exist/work:**

(a) Decomposition

(b) Pasteurisation

(c) Steamer

(d) Refrigerator

(e) Deep freezer

(f) Human body

Based on the information provided in the picture, predict the chances of growth and survival of microbes in each of the above.